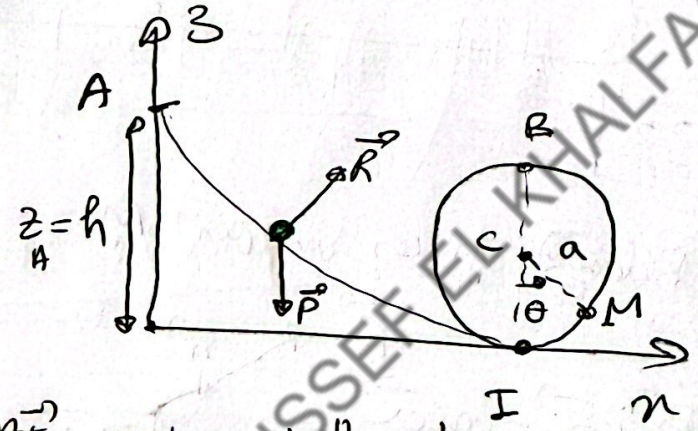
CONCOURS ENSAM – 2023

Beinscience

Partie B

32 V_M ?



- Théorème de E_c entre A et I

$$\Delta E_{c\ A\to I} = W_{A\to I}(\vec{P}) + W_{A\to I}(\vec{R})$$

$+ W_{A\to I}(\vec{R}) = \vec{R}\cdot\vec{AI} = 0$ car $\vec{R} \perp \vec{AI}$ mvt sans frottement

$+ W_{A\to I}(\vec{P}) = mg(z_A - z_I) = mg(h - 0) = mgh$

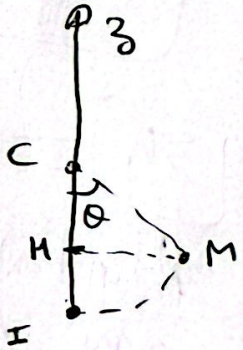
$+ \Delta E_{c\ A\to I} = \frac{1}{2}mV_I^2 - \underbrace{\frac{1}{2}mV_A^2}_{0} = \frac{1}{2}mV_I^2$

$\rightarrow \frac{1}{2}mV_I^2 - 0 = mgh + 0 \Rightarrow \boxed{V_I = \sqrt{2gh}}$

- Théorème de E_c entre I et M

$$\Delta E_{c\ I\to M} = W_{I\to M}(\vec{P}) + W_{I\to M}(\vec{R})$$

$+ W_{I\to M}(\vec{R}) = 0$; $W_{I\to M}(\vec{P}) = mg(z_I - z_M) = -mgz_M$;



$z_M = IH = IC - HC = a - a\cos\theta = a(1 - \cos\theta)$

$\Rightarrow W(\vec{P}) = -mga(1 - \cos\theta) = mga(\cos\theta - 1)$

$\rightarrow \frac{1}{2}mV_M^2 - \frac{1}{2}mV_I^2 = mga(\cos\alpha - 1)$

$$\frac{1}{2}mv_M^2 = \frac{1}{2}m(2gh) + mga(\cos\alpha - 1)$$

$$\boxed{v_M = \sqrt{2g\left(h + a(\cos\alpha - 1)\right)}}$$

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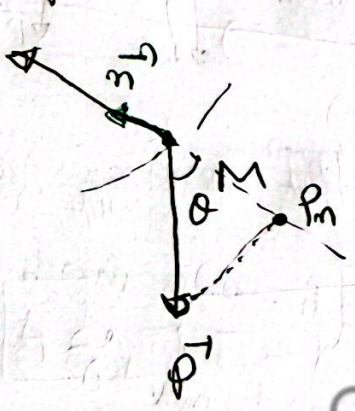
Bilan des forces au point M



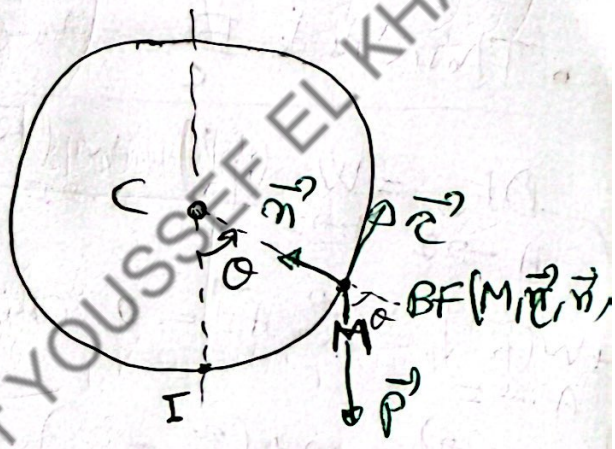
$$\vec{P} + \vec{R} = m\vec{a}$$

proj $|\vec{n}$:

$$P_n + R_n = ma_n$$

$$-mg\cos\alpha + R = m\frac{v_M^2}{a}$$

$$\boxed{R = m\left(\frac{v_M^2}{a} + g\cos\alpha\right) \geq 0}$$



$$\vec{a} = \vec{a}_n + \vec{a}_\tau$$

$$a_n = \frac{v_M^2}{a} \leftarrow \text{rayon}$$

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L'intensité R doit rester positive pour maintenir le contact entre M et le cercle $R > 0$

$$R > 0 \Rightarrow m\left(\frac{v_M^2}{a} + g\cos\alpha\right) > 0 \Rightarrow m\left(\frac{2g\left(h + a(\cos\alpha - 1)\right)}{a} + g\cos\alpha\right) >$$

$$\frac{2h}{a} + 2(\cos\alpha - 1) + \cos\alpha > 0$$

$$\Rightarrow \boxed{h > \frac{a}{2}\left(2 - 3\cos\theta\right)}$$

$\theta = \pi \Rightarrow \cos\pi = -1 \Rightarrow h > \frac{a}{2}\left(2 - 3(-1)\right)$

$\Rightarrow h \geq \frac{5a}{2} = h_{min}$

$$\boxed{h_{min} = \frac{5a}{2}}$$

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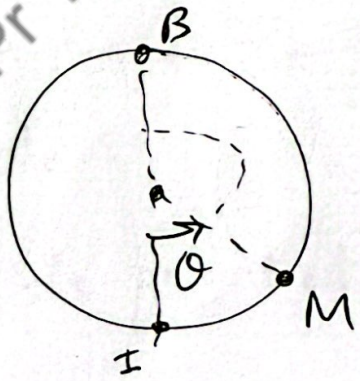
$$V_M = V_M(\theta) = \sqrt{2g\left(h + a(\cos\theta - 1)\right)}$$

$M \longrightarrow B$ si $\theta = \pi$

$$V_B = V_M(\theta = \pi) = \sqrt{2g\left(h_{min} + a(\cos\pi - 1)\right)}$$

$$= \sqrt{2g\left(\frac{5a}{2} + a(-1-1)\right)}$$



$$\boxed{V_B = \sqrt{g \cdot a}}$$

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La réaction au point I ($M \longmapsto I$; si $\theta = 0$)

$$R_I = R(\theta = 0) = m\left(\frac{V_I^2}{a} + g\cos(0)\right) = m\left(\frac{2gh_{min}}{a} + g\right)$$

$$R_I = m\left(\frac{2g}{a} \cdot \frac{5a}{2} + g\right) = +6mg \Rightarrow \boxed{R_I = 6mg}$$