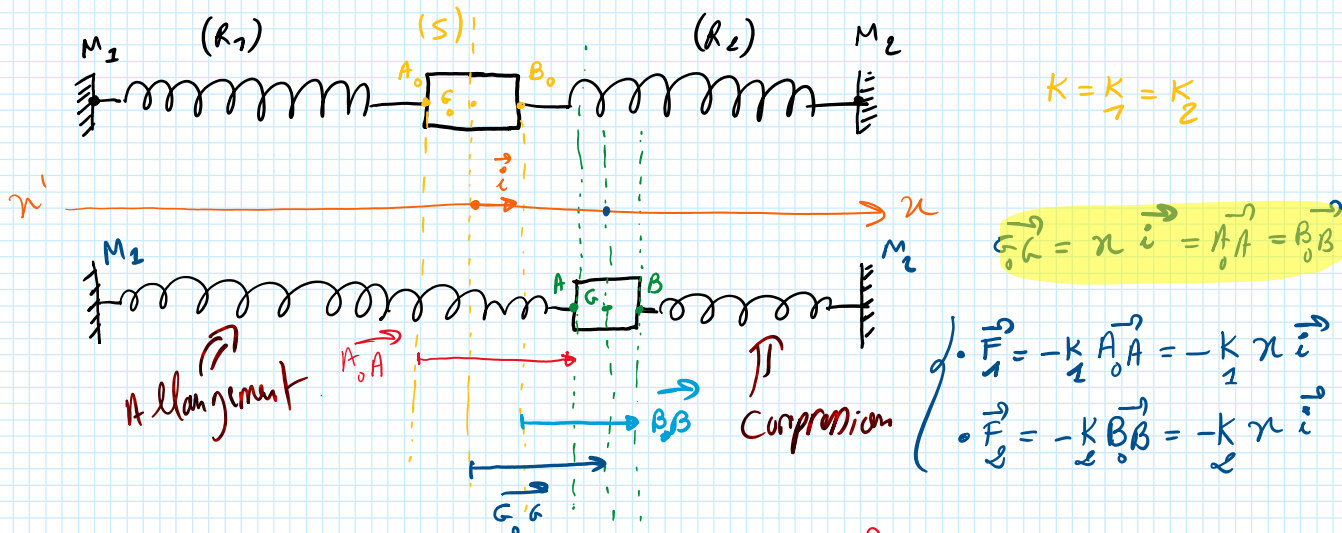


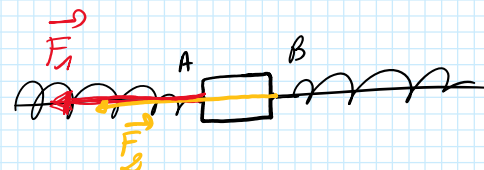


Oscillateur avec deux Ressorts

I

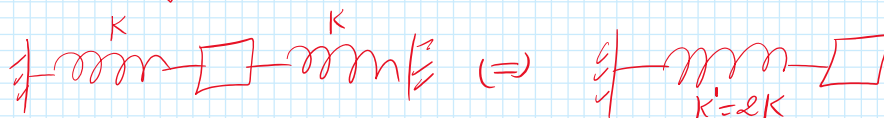


$K = K_1 = K_2 \Rightarrow \vec{F}_1 = \vec{F}_2 = -k x \vec{i}$



$\vec{P} + \vec{R} + \vec{F}_1 + \vec{F}_2 = m \vec{a}_c \Rightarrow \text{proj/on } : -kx - kx = m \ddot{x}$

$\Rightarrow \ddot{x} + \frac{2k}{m} x = 0 \Rightarrow \ddot{x} + \frac{k'}{m} x = 0$



$\omega_0^2 = \frac{2k}{m} = \frac{k'}{m} = \frac{k_1 + k_2}{m}$

$T_0 = 2\pi \sqrt{\frac{m}{k'}} = 2\pi \sqrt{\frac{m}{2k}} = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$

$E_c = \frac{1}{2} m \dot{x}^2 ; E_{pe} = E_{pe1} + E_{pe2} = \frac{1}{2} k x^2 + \frac{1}{2} k x^2 + ct = kx^2 + ct$

$E_m = \frac{1}{2} m \dot{x}^2 + kx^2 + ct = \text{Constante}$

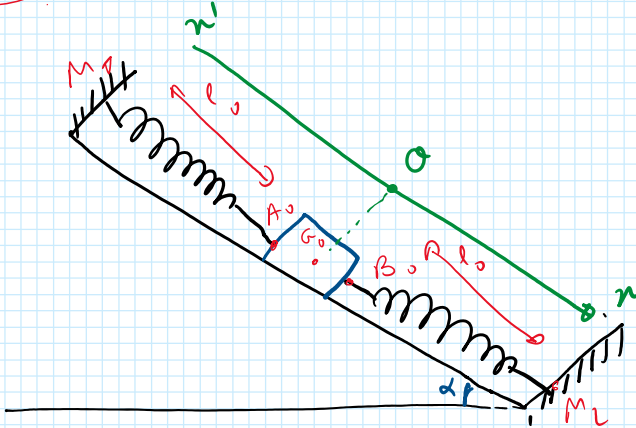
$$\frac{dE_m}{dt} = 0 \Rightarrow m\ddot{x} + 2kx\dot{x} = 0$$

$$\dot{x} \left( \underbrace{m\ddot{x} + 2kx}_{=0} \right) = 0$$

$$\ddot{x} + \frac{2k}{m}x = 0$$



on incline l'ensemble d'un angle  $\alpha$  / horizontal



$$\vec{A}_0A'_0 = \vec{B}_0B'_0 = \vec{G}_0G'_0 = x_0 \vec{i} = \Delta l_0 \vec{i}$$

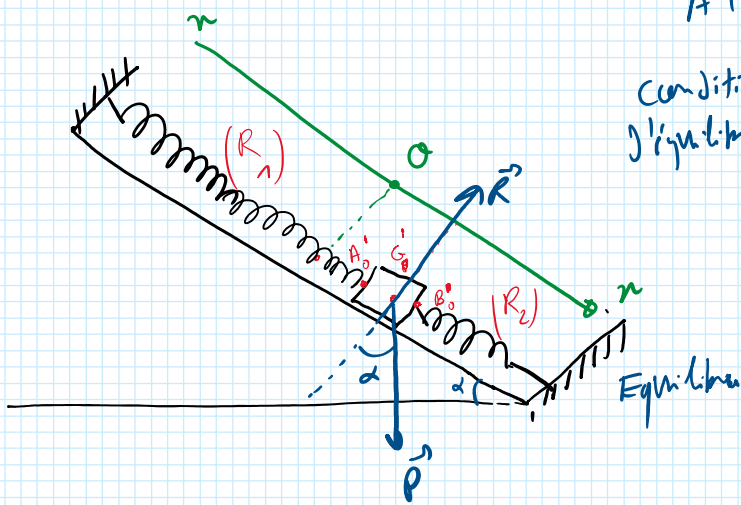
$$\left. \begin{aligned} \vec{F}_{1_0} &= -k_1 \vec{A}_0A'_0 = -k x_0 \vec{i} \\ \vec{F}_{2_0} &= -k_2 \vec{B}_0B'_0 = -k x_0 \vec{i} \end{aligned} \right\} \vec{F}_{1_0} = \vec{F}_{2_0} = \vec{F}_{\Delta_0}$$

A l'équilibre :  $\vec{p} + \vec{R} + \vec{F}_{1_0} + \vec{F}_{2_0} = \vec{0}$

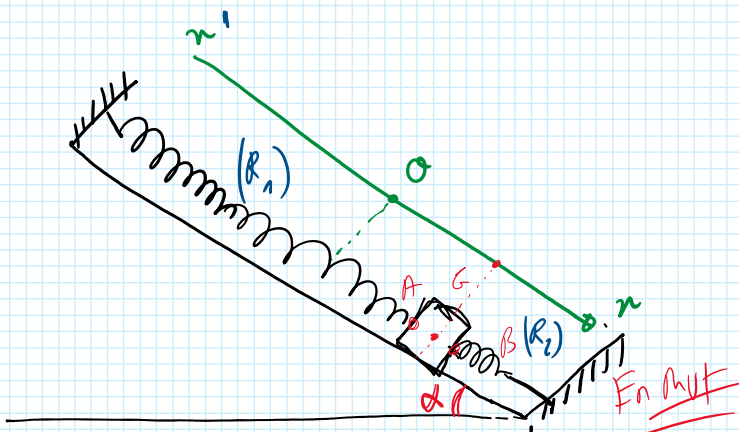
condition d'équilibre  $\rightarrow$

$$mg \sin \alpha - 2kx_0 = 0$$

$$x_0 = \Delta l_0 = \frac{mg \sin(\alpha)}{2k}$$



Equilibre



En mouvement

$$\vec{A}_0A = \vec{B}_0B = \vec{G}_0G = (x_0 + x) \vec{i}$$

$$\left. \begin{aligned} \vec{F}_1 &= -k_1 \vec{A}_0A = -k(x_0 + x) \vec{i} \\ \vec{F}_2 &= -k_2 \vec{B}_0B = -k(x_0 + x) \vec{i} \end{aligned} \right\}$$

$$\vec{p} + \vec{R} + \vec{F}_1 + \vec{F}_2 = m \vec{a}$$

$$mgsin\alpha + 0 - 2k(x_0 + n) = m\ddot{n}$$

$$\Rightarrow \underbrace{mgsin\alpha - 2kx_0}_{0} - 2kn = m\ddot{n}$$

$$- 2kn = m\ddot{n} \Rightarrow \ddot{n} + \frac{2k}{m}n = 0$$

$$\ddot{n} + \frac{k'}{m}n = 0 \quad \text{avec } k' = 2k$$

$$\omega_0^2 = \frac{2k}{m} = \frac{k_1 + k_2}{m}$$

$$\bullet E_{pp} = -mgs + c_1 = -mgnsin\alpha + c_2$$

$$\bullet E_{pe} = E_{pe1} + E_{pe2}$$

$$\vec{A}_0A = \vec{B}_0B = \vec{G}_0\vec{h} = (x_0 + n)\vec{i}$$

$$\vec{F}_1 = \vec{F}_2$$

$$Dl_1 > 0 \text{ et } Dl_2 < 0 \Rightarrow \underbrace{Dl_1 \vec{i}}_{\vec{F}_1} = - \underbrace{Dl_2 \vec{i}}_{\vec{F}_2} \Rightarrow Dl_1 = -Dl_2 = x_0 + n$$

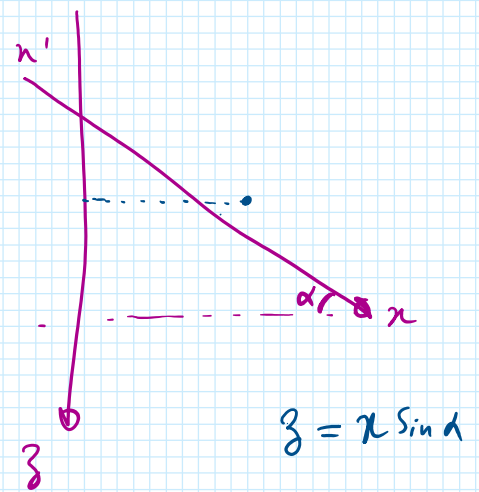
$$\Rightarrow Dl_1 = x_0 + n \quad \text{et} \quad Dl_2 = -x_0 - n = \underbrace{l_1}_{l_1} - \underbrace{l_0}_{l_0} = -(x_0 + n)$$

$$E_{pe1} = \frac{1}{2}k(Dl_1)^2 + c_2 = \frac{1}{2}k(x_0 + n)^2 + c_2$$

$$E_{pe2} = \frac{1}{2}k(Dl_2)^2 + c_3 = \frac{1}{2}k(x_0 + n)^2 + c_3$$

$$E_p = -mgnsin\alpha + k(x_0 + n)^2 + c_4$$

$$E = \frac{1}{2}m\dot{n}^2 - mgnsin\alpha + k(x_0 + n)^2 + c_4 = \text{Constante}$$



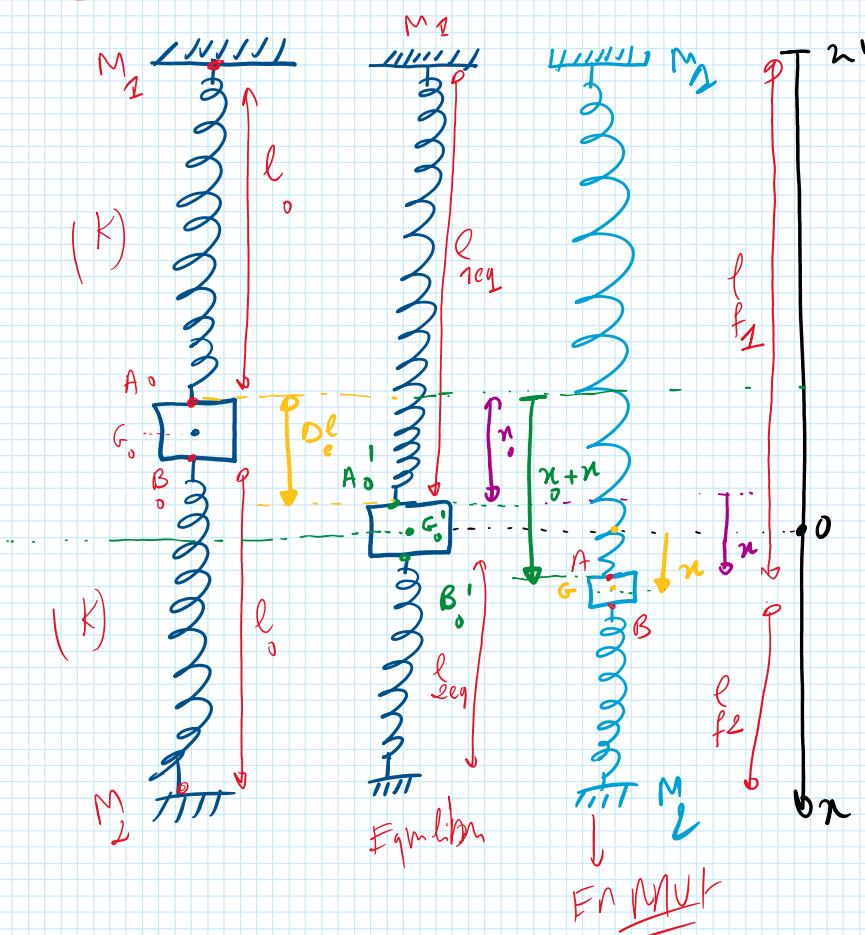
$$E_m = \frac{1}{2} m \dot{x}^2 - mgx \sin \alpha + k(x_0 + x)^2 + ct = \text{constab}$$

$$\frac{dE_m}{dt} = 0 \Rightarrow m \dot{x} \ddot{x} - mg \dot{x} \sin \alpha + 2k(x_0 + x) \dot{x} = 0$$

$$m \ddot{x} - \underbrace{mg \sin \alpha + 2kx_0 + 2kx}_0 = 0$$

$$\ddot{x} + \frac{2k}{m} x = 0$$

III



$$\vec{G}_0 = \vec{k}_0' = \vec{A}_0 \vec{A}_0' = \vec{B}_0 \vec{B}_0' = \Delta l_e \vec{i}$$

$$\vec{F}_{T0} = -k \vec{A}_0 \vec{A}_0' = -k \Delta l_e \vec{i}$$

$$\vec{F}_{T0} = -k \vec{B}_0 \vec{B}_0' = -k \Delta l_e \vec{i}$$

$$\vec{p} + \vec{F}_{T0} + \vec{F}_{T0} = \vec{0}$$

$$mg - k \Delta l_e - k \Delta l_e = 0$$

$$mg = 2k \Delta l_e = 2k x_0$$

$$\Delta l_e = \frac{mg}{2k} = x_0 > 0$$

$$\Delta l_{req} = l_{req} - l_0 > 0$$

$$\Delta l_{2eq} = l_{2eq} - l_0 < 0$$

$$\Delta l_{req} = l_{req} - l_0 = \Delta l_e = x_0 = mg/2k$$

$$\Delta l_{2eq} = l_{2eq} - l_0 = -\Delta l_e = -x_0 = -mg/2k$$

$$0 \quad 0 \quad - \quad 0 \quad 0 \quad 0 \quad 0 = (l - l_0) - (l_0 - l_0)$$

$$\begin{aligned}
 l_{1eq} - l_{2eq} &= l_{1q} - l_0 + l_0 - l_{2q} = \underbrace{(l_{1q} - l_0)}_{x_0} - \underbrace{(l_{2q} - l_0)}_{-x_0} \\
 &= 2x_0 = 2Dl_e = 2 \cdot \frac{mg}{2K} = \frac{mg}{K}
 \end{aligned}$$

En Mvt

$$\vec{A}_0 \vec{A} = \vec{B}_0 \vec{B} = \vec{r}_0 \vec{x} = (Dl_e + x) \vec{i} = (x_0 + x) \vec{i}$$

$$\vec{F}_1 = -K_1 \vec{A}_0 \vec{A} = -K(x_0 + x) \vec{i}$$

$$\vec{F}_2 = -K_2 \vec{B}_0 \vec{B} = -K(x_0 + x) \vec{i}$$

PFD

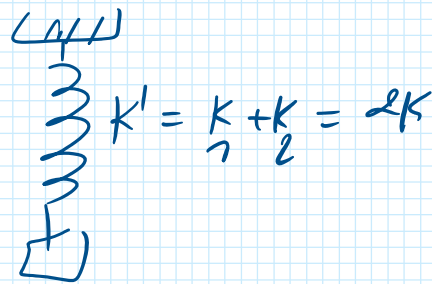
$$\vec{P} + \vec{F}_1 + \vec{F}_2 = m\vec{a} \Rightarrow mg - 2K(x_0 + x) = m\ddot{x}$$

$$\underbrace{mg - 2Kx_0}_0 - 2Kx = m\ddot{x}$$

$$\Rightarrow \ddot{x} + \frac{2K}{m} x = 0 \Leftrightarrow \ddot{x} + \frac{K'}{m} x = 0$$



(=)



$$\vec{A}_0 \vec{A} = \vec{B}_0 \vec{B}$$

$$\vec{F}_1 = \vec{F}_2 \Rightarrow Dl_1 \vec{i} = -Dl_2 \vec{i} \quad \left| \begin{array}{l} l_1 - l_0 = Dl_1 > 0 \\ l_2 - l_0 = Dl_2 < 0 \end{array} \right.$$

$$\left. \begin{aligned} \Delta l_1 &= \Delta l_e + n = n_0 + n \\ \Delta l_2 &= -(\Delta l_e + n) = -(n_0 + n) \end{aligned} \right\} \begin{aligned} E_{pe} &= E_{pe1} + E_{pe2} \\ &= \frac{1}{2} K (n_0 + n)^2 + \frac{1}{2} K (n_0 + n)^2 + \Delta l_1 \\ \circ E_{pe} &= K (n_0 + n)^2 + \Delta l_1 \\ \circ E_{pp} &= -m g n + \Delta l_2 \end{aligned}$$

$$E_m = \frac{1}{2} m \dot{n}^2 - m g n + K (n_0 + n)^2 + \Delta l_1 = \text{Constante}$$

$$\frac{\partial E_m}{\partial t} = 0 \Rightarrow$$

$$\begin{aligned} \ddot{n} + \frac{2K}{m} n &= 0 \\ \ddot{n} + \frac{K_1 + K_2}{m} n &= 0 \end{aligned}$$