Concours Marrakech 2003

Q1 :

متتالية حسابية تناقصية أساسها r

بحيث :

$4(u_1)^2 + (u_2)^2 = 164$

لدينا : $u_n = u_p + (n-p)r$

اذن : $4(u_0 + r)^2 + (u_0 + 2r)^2 = 164$

$\Rightarrow 16 + 16r + 4r^2 + 4 + 8r + 4r^2 = 164$

$\Rightarrow r^2 + 3r - 18 = 0$

$\Delta = 81$

$r_1 = 3 \; ; \; r_2 = -6$

بما أن u_n تناقصية فإن

$u_{n+1} - u_n < 0$

اذن $r < 0$

اذن $r = -6 \Leftarrow$ B

Q2

(u_n) هندسية أساسها $q > r$

بحيث $u_9 = 1280$ و $u_1 = 5$

$q = ?$

$u_9 = q^{9-1} u_1$

$1280 = q^8 \, 5$

$q^8 = \frac{1280}{5} \Rightarrow q = 2 \Rightarrow$ D

Q3

$\lim S_n$

$S_n = 1 + \frac{1}{2} + \dots + \frac{1}{2^n}$

$S_n = 1 \times \left(\frac{1 - (\frac{1}{2})^{n+1}}{1 - \frac{1}{2}} \right)$

$S_n = 1 \times \left(\frac{1 - (\frac{1}{2})^{n+1}}{1/2} \right)$

$S_n = 2\left(1 - \left(\frac{1}{2}\right)^{n+1}\right)$

$\Rightarrow \lim_{n \to +\infty} S_n = 2(1-0) = 2$

لأن $-1 < \frac{1}{2} < 1$

Q4 $f(x) = \frac{x^2 - \ln x}{x - 1}$

$= \frac{x^2 - 1 + 1 - \ln x}{x - 1}$

$= x + 1 + \frac{1 - \ln x}{x - 1}$

$= x + 1 + \frac{x(\frac{1}{x} - \frac{\ln x}{x})}{x(1 - \frac{1}{x})}$

الفكرة هي :

كتابة دالة على شكل

$f(x) = ax + b + g(x)$

مع $\lim_{+\infty} g(x) = 0$

Q5

$x_0 = ?$ بحيث $f'(x_0) = x_0$

$x_0^2 + 2x_0 - 3 = 0$

$\Delta = 16 \Rightarrow x_0 = 1$

$x_0 = -3$

$A(-3 \; ; \; f(-3) = 9)$

Q7

$4^3 = 64$

Q6

$P(A) = \frac{C_2^2 + C_3^2}{C_5^2} = \frac{2}{5}$

Q2 Kech 2004

$b = aq$

$c = bq$

$\begin{cases} b/q + b + qb = 20 \\ bq - \frac{b}{q} = 100 \end{cases}$

$$\begin{cases} a + b - c = 20 \\ c - a = 100 \end{cases}$$

$$\begin{cases} b - 100 = 20 \Rightarrow b = 120 \\ a - c = -100 \end{cases}$$

$120q - \dfrac{120}{q} = 100$

$\Rightarrow 120q^2 - 120 = 100$

$\Rightarrow 120q^2 - 220 = 0$

$\Rightarrow 30q^2 - 55 = 0$

$\Delta =$

$\Rightarrow q_1 =$

$q_2 =$

(U_n) م. حسابية

$U_n = U_p + (n-p)r$

(U_n) م. هندسية

$U_n = q^{n-p} U_p$

$$\lim_{n \to +\infty \text{ ou } 0^+} \frac{a\ln(n) + b}{c\ln(n) + d} = \frac{a}{c}$$

C_f لـ مقارب مائل $y = ax + b$

$\Rightarrow$
- $\lim_{x \to \pm\infty} f(x) = \infty$
- $\lim_{x \to \pm\infty} \frac{f(x)}{x} = a$
- $\lim_{x \to \pm\infty} f(x) - ax = b$

اذا كان S_n هو مجموع عدد حدود متتابعة لمتتالية حسابية بحيث فإن :

$S_n = \dfrac{\text{عدد حدود } S_n}{2}$ (الحد الأول لـ S_n + الحد الأخير لـ S_n)

$(1+i)^2 = 2i \quad \Big\| \quad (1-i)^2 = -2i$

Cnens Kech 2005

Q_1

(U_n) م.ح

$U_1 + U_2 + \dots + U_{20} = ?$

$U_1 = 5 \;;\; U_2 = 8$

$U_2 = U_1 + (2-1)r$

$r = U_2 - U_1 = 3$

$U_{20} = U_1 + (20-1)r$

$U_{20} = 5 + 19 \times 3$

$= 62$

$U_1 + U_2 + \dots + U_{20} = \frac{20}{2}(U_1 + U_{20})$

$= 10(5 + 62)$

$= 670 \Rightarrow E$

Q_2

(U_n) م.ه حيث

$U_4 = 60$ و $U_2 = 15$

$q = ?$ $\quad (U_n = q^{n-p} U_p)$

$(q > 0)$

$U_4 = q^{4-2} U_2$

$q^2 = 4$

$q = 2 \Rightarrow A$

$$\lim_{x\to 0^+} \frac{\ln x}{1-\ln x} = \lim_{x \to 0^+} \frac{\ln x}{\ln x\left(\frac{1}{\ln x}-1\right)} \quad : Q_3$$

$= -1$

Q_4

$$f(x) = \frac{2x^2 - 3x + \ln x}{x}$$

مقارب مائل $y = ax + b$

$$\begin{cases} f(x) = ax + b + g(x) \\ \lim_{x\to+\infty} g(x) = 0 \end{cases}$$

$$f(x) = \frac{2x^2 - 3x}{x} + \frac{\ln x}{x}$$

$$f(x) = 2x - 3 + \frac{\ln x}{x}$$

$y = 2x-3$ مقارب مائل لـ $\mathcal{C}$ بجوار ∞

Q_5

$\ln(x+2) + \ln(x+3) = \ln(6)$

$\ln(x+2)(x+3) = \ln 6$

$x^2 + 5x + 6 = 6$

$x(x+5) = 0$

$x = 0$ أو $x = -5$

$\Rightarrow \notin D_f$

$D_f =]-5, +\infty[$

$S = \{0\}$

Q_6

$z = 2 + \frac{2\sqrt{3}}{3} i$

$|z| = \sqrt{2^2 + \left(\frac{2\sqrt{3}}{3}\right)^2} = \sqrt{4 + \frac{12}{9}}$

$= \frac{4\sqrt{3}}{3}$

$z = \frac{4\sqrt{3}}{3}\left(2 \times \frac{3}{4\sqrt{3}} + \frac{2\sqrt{3}}{3} \times \frac{3}{4\sqrt{3}} i\right)$

$= \frac{4\sqrt{3}}{3}\left[\frac{\sqrt{3}}{2} + \frac{1}{2} i\right)$

$= \frac{4\sqrt{3}}{3}\left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right)$

$= \left[\frac{4\sqrt{3}}{3}, \frac{\pi}{6}\right] \Rightarrow$ D

Q_7

$$\left(\frac{1-i}{1+i}\right)^{16} = \frac{(1-i)^{16}}{(1+i)^{16}} = \left(\frac{(1-i)^2}{(1+i)^2}\right)^{8}$$

$= \left(\frac{-2i}{2i}\right)^8 = (-1)^8 = 1$

$\Rightarrow$ B

Q_8

$-1x + 2y + 3z -$

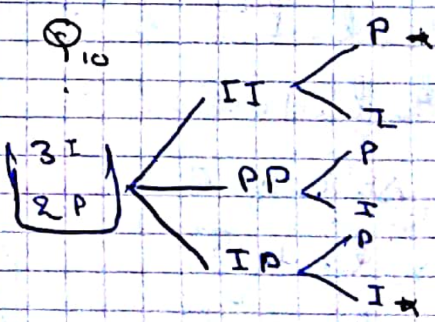
Q_9

B	B	
N	N	N

$\text{card}(A) = C_2^1 \times C_3^1 = 6$

$p(A) = \frac{\text{card}(A)}{\text{card}(\Omega)} = \frac{6}{C_5^2} = \frac{6}{10} = \frac{3}{5}$

Q_{10}



$$p = \frac{C_3^2}{C_5^2} \times \frac{C_2^1}{C_5^1} + \frac{C_3^1 \times C_2^1}{C_5^2} \times \frac{C_3^1}{C_5^1}$$

$= \frac{3}{10} \times \frac{2}{5} + \frac{6}{10} \times \frac{3}{5}$

$\Rightarrow \frac{12}{25} \Rightarrow$ B

Concours Kech 2006

Q_1

$$\begin{cases} U_0 = 1 \\ U_4 = 13 \end{cases} \quad \text{S.A } (U_n)$$

$U_{10} = ?$

$U_4 = U_0 + 4r$

$13 = 1 + 4r$

$\Rightarrow 4r = 12 \Rightarrow r = 3$

$U_{10} = U_0 + 10r$

$U_{10} = 1 + 10 \times 3 = 31$

$U_{10} = 31 \Rightarrow$ D

Q_2

$$\begin{cases} U_{n+1} = \sqrt{1 + U_n} \\ U_0 = 0 \end{cases}$$

$\lim\limits_{n \to \infty} U_n = ?$

$f(x) = \sqrt{1+x}$

$f(l) = l$

$\sqrt{1+l} = l$

$1 + l = l^2 \quad (l \geq 0)$

$l^2 - l - 1 = 0$

$\Delta = 1 + 4 = 5$

$\Rightarrow l = \frac{1+\sqrt{5}}{2}$

$l_2' = \frac{1-\sqrt{5}}{2} \notin$ $l \geq 0$

$\Downarrow$
B

Q_3:

$f(x) = 2x^2 - 1 \; ; \; x \leq 1$

$f(x) = \alpha \frac{\sin(x^2-1)}{x-1} \; ; \; x > 1$

قيمة α لـ f متصلة على $\mathbb{R}$

~~سانت f متصلة على كل عنصر~~

سا أن f متصلة على $\mathbb{R}$

فهي متصلة على كل عدد من $\mathbb{R}$

$\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^-} f(x)$

$\lim\limits_{x \to 1^+} \alpha \frac{\sin(x^2-1)}{x-1} = \lim\limits_{x \to 1^-} 2x^2 - 1$

$\lim\limits_{x \to 1^+} \frac{\alpha \sin(x^2-1)}{x^2-1}(x+1) = \lim\limits_{x \to 1^-} 2x^2 - 1$

$\alpha \times 1 (1+1) = 1$

$2\alpha = 1$

$\alpha = \frac{1}{2} \Rightarrow$ C

Q_4 $f: I = [1, +\infty[\longrightarrow f(I)$

$x \longmapsto f(x) = \frac{(x-1)^2}{x}$

$f^{-1}(1) = ?$

$f^{-1}: f(I) \to I = [1, +\infty[$

$x \to f(x)$

$1 \to f^{-1}(1)$

$f(x) = y$

$f(y) = x$

$f^{-1}(1) = y \Leftrightarrow f(y) = 1$

$\Leftrightarrow \frac{(y-1)^2}{y} = 1$

$\Leftrightarrow y^2 - 2y + 1 = y$

$\Rightarrow y^2 - 3y + 1 = 0$

$\Delta = 9 - 4 = 5$

$y = \frac{3-\sqrt{3}}{2} \simeq 0,4 \notin I$

$y = \frac{3+\sqrt{3}}{2} \simeq 2,6 \in I$

Q_5

$f(x) = \sqrt{\frac{1-\sin x}{1+\sin x}}$

$D_f = ?$

$-1 \leq \sin x \leq 1$

$-1 \leq \sin x \leq 1 \Rightarrow 0 \leq 1+\sin x \leq 2$

$-1 \leq -\sin x \leq 1 \Rightarrow 0 \leq 1-\sin x \leq 2$

$x \in D_f \Leftrightarrow 1+\sin x \neq 0$

$\Leftrightarrow \sin x \neq -1$

$\Leftrightarrow x \neq -\frac{\pi}{2} + 2k\pi$

$\Leftrightarrow x \neq \frac{3\pi}{2} + 2k\pi$

$D_f = \mathbb{R} - \left\{\frac{3\pi}{2} + 2k\pi\right\}$

$\sin x = 0 \Rightarrow x = k\pi$

$\sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + 2k\pi$

$\sin x = -1 \Leftrightarrow x = -\frac{\pi}{2} + 2k\pi$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}$

Q_6

$f(x) = \frac{1-\sqrt{1-x^2}}{x}$, $x \in [-1,0[\cup]0,1]$

$f(0) = 0$

$f'(0) = ?$

$f'(x_0) = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$

$f'(x_0) = \lim_{x \to 0} \frac{f(x)-f(0)}{x-0}$

$= \lim_{x \to 0} \frac{\frac{1-\sqrt{1-x^2}}{x} - 0}{x}$

$= \lim_{x \to 0} \frac{1-\sqrt{1-x^2}}{x^2}$

$= \lim_{x \to 0} \frac{1^2 - (\sqrt{1-x^2})^2}{x^2(1+\sqrt{1-x^2})}$

$= \lim_{x \to 0} \frac{1-(1-x^2)}{x^2(1+\sqrt{1-x^2})}$

$= \lim_{x \to 0} \frac{1-1+x^2}{x^2(1+\sqrt{1-x^2})}$

$= \frac{1}{1+\sqrt{1+0}} = \frac{1}{2} \Rightarrow$ (E)

Q_7 :

$x \log x^2 + \log 2 = 0$

نعوض بالحلول المقترحة

Q_8

جذر مربع لـ Z : $z = x+iy$

$\Leftrightarrow z^2 = Z$

$\Leftrightarrow \begin{cases} x^2+y^2 = |Z| & (1) \\ x^2-y^2 = Re(Z) & (2) \\ xy = \frac{Im(Z)}{2} & (3) \end{cases}$

$(1)+(2) \Leftrightarrow 2x^2 = |Z| + Re(Z)$

$(1)-(2) \Leftrightarrow 2y^2 = |Z| - Re(Z)$

et d'après (3) on fait le choix

خاصية :

لكل عدد عقدي جذرين مربعين متقابلين

$$\begin{cases} x^2+y^2 = |3+4i| \\ x^2-y^2 = Re(3+4i) \\ xy = \dfrac{Im(3+4i)}{2} \end{cases}$$

نضع $z = x+iy$

$$\begin{cases} x^2+y^2 = 5 & (1) \\ x^2-y^2 = 3 & (2) \\ xy = 2 & (3) \end{cases}$$

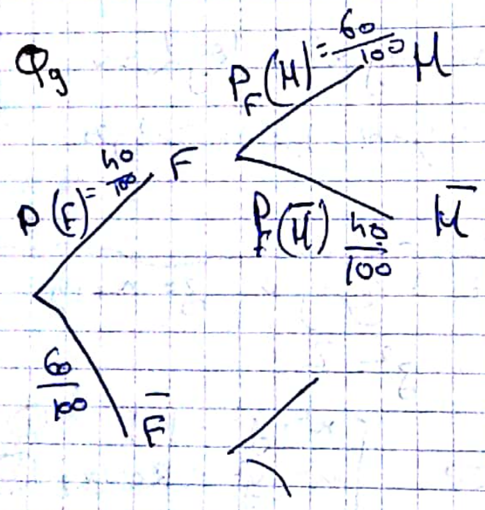
$(1)+(2) \iff 2x^2 = 8 \iff x^2 = 4 \iff x = 2$ أو $x = -2$

$(1)-(2) \iff 2y^2 = 2 \iff y^2 = 1 \iff y = 1$ أو $y = -1$

d'après on a $xy > 0 \to z_1 = 2+1i$, $z_2 = -2-i$

$S = \{2+i, -2-i\}$

Q9



$$P_M(F) = \frac{P(F \cap M)}{P(M)} = \frac{P(F) \times P_F(M)}{P(M)}$$

$$= \frac{\frac{40}{100} \times \frac{60}{100}}{\frac{30}{100}}$$

Q_{10}

$f(x) = \sqrt[3]{8 - x^3}$

$x \in D_f \iff 8 - x^3 \geq 0$

$\iff 2^3 - x^3 \geq 0$

$\iff (2-x)(4 + 2x + x^2) \geq 0$

$\iff 2 - x \geq 0$

$\iff -x \geq -2$

$\iff x \leq 2$

$\iff x \in]-\infty, 2]$

$f^{-1}(x) = ?$

$f : \mathbb{R} \longrightarrow \mathbb{R}$

$x \longrightarrow f^{-1}(x) = y$

$f^{-1}(x) = y \iff f(y) = x$

$\iff \sqrt[3]{8 - y^3} = x$

$\iff 8 - y^3 = x^3$

$\iff y^3 = 8 - x^3$

$\iff y = \sqrt[3]{8 - x^3}$

" Concours Kech 2007 "

Q_1

$\begin{cases} U_2 = 3 \\ U_5 = -24 \end{cases}$

s.g (U_n)

$U_n = q^{n-p}\, U_p$

$U_5 = q^{5-2}\, U_2$

$U_5 = q^3\, U_2$

$q^3 = -8$

$q = -2 \Rightarrow$ C

Q_2

$\lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2+3x} - \sqrt{x^2+1}}{x} + 1$

$= \lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2+3x}^2 - \sqrt{x^2+1}^2}{x\left(\sqrt{x^2+3x} + \sqrt{x^2+1}\right)} + 1$

$= \lim\limits_{x \to +\infty} \dfrac{x^2 + 3x - x^2 + 1}{x\left(\sqrt{x^2+3x} + \sqrt{x^2+1}\right)} + 1 = 1$

$\Rightarrow$ B

Q_3

$F(x) = \sqrt{x^2+3x} - \sqrt{x^2+3x}$

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \dfrac{3x - 1}{\sqrt{x^2+3x} + \sqrt{x^2+1}}$

$= \lim\limits_{x \to +\infty} \dfrac{x\left(3 - \frac{1}{x}\right)}{x\left(\sqrt{1+\frac{3}{x}} + \sqrt{1+\frac{1}{x}}\right)} = \dfrac{3}{2}$

$\Rightarrow$

$y = x + \dfrac{3}{2} \Rightarrow$ A

Q_4

$z = \left(\dfrac{1+i}{1-i}\right)^2 = \dfrac{(1+i)^2}{(1-i)^2} = \dfrac{2i}{-2i} = -1$

$\Rightarrow$ B

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Q5

(V_n) م. حسابية.

$V_n = V_p + (n-p)r$

$V_4^2 + V_2^2 = 10$

$V_0 = 1$

$r = ?$

$(V_0 + 4r)^2 + (V_0 + 2r)^2 = 10$

$(1+4r)^2 + (1+2r)^2 = 10$

$1 + 8r + 16r^2 + 1 + 4r + 4r^2 - 10 = 0$

$-8 + 12r - 20r^2 = 0$

$5r^2 + 3r - 2 = 0$

$\Delta = 9 + 40 = 49$

بما أن $r > 0$: -1 ، $\frac{2}{5}$

$r = \frac{2}{5} \Rightarrow$ C

Q6

$$\begin{cases} w_0 = \frac{1}{2} \\ w_{n+1} = -1 - \frac{1}{4w_n} = f(w_n) \end{cases} \quad / \; f(x) = -1 - \frac{1}{4x}$$

$f(l) = l$

$-1 - \frac{1}{4l} = l$

$-4l - 1 = 4l^2$

$4l^2 + 4l + 1 = 0$

$(2l+1)^2 = 0$

$2l + 1 = 0$

$l = -\frac{1}{2} \Rightarrow$ D

Q7 $3e^{2x} - 4e^x + 1 = 0$

$\Delta = (-4) - 4\times 3 = 4$

$$\begin{cases} x = -\ln 3 \\ x = 0 \end{cases} \Leftarrow \begin{cases} e^x = \frac{4-2}{6} = \frac{1}{3} \\ e^x = \frac{4+2}{6} = 1 \end{cases}$$

Q8

$$f(x) \begin{cases} f(x) = \frac{x\ln(1+3x)}{1-\cos 2x} + a \; ; \; x \in]0,1] \\ f(x) = x + \frac{1}{2} \; ; \; x \in]-\frac{1}{3}, 0] \end{cases}$$

f مستمرة عند 0 $\Leftrightarrow$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$\lim_{x\to 0^-} \frac{x\ln(1+3x)}{1-\cos 2x} + a = \lim_{x\to 0^+} x + \frac{1}{2}$

$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1 \; ; \; \lim_{x\to 0} \frac{1-\cos x}{x} = \frac{1}{2}$

$\lim_{x\to 0^-} \frac{\ln(1+3x)}{3x} \times \frac{(2x)^2}{1-\cos 2x} \times \frac{3}{4} + a = \frac{1}{2}$

$1 \times 2 \times \frac{3}{4} + a = \frac{1}{2}$

$a = \frac{1}{2} - \frac{3}{2} \Rightarrow a = -1$

$\Downarrow$

D

Q9

		$e^{-\frac{b}{a}}$	
$a\ln(x)+b$	عكس إشارة a	0	إشارة a

$g(x) = \frac{x}{\sqrt{4-(\ln x)^2}}$

$x \in D_g \Leftrightarrow x > 0$ و $4 - (\ln(x))^2 > 0$

$\Leftrightarrow (2 - \ln(x))(2 + \ln(x)) > 0$

x		e^{-2}		e^2	
$2-\ln(x)$	+		+	0	−
$2+\ln(x)$	−	0	+		+
الإشارة	−		+		−

$D_f =]e^{-2}, e^2[$

$\Downarrow$

C

Q_{10}

$$h(x) = \begin{cases} h(x) = \dfrac{\sqrt{x^2+2} - \sqrt{2}}{x} \\ h(0) = 0 \end{cases}$$

$$h'(0) = \lim_{x \to 0} \frac{h(x) - h(0)}{x - 0} = \lim_{x \to 0} \frac{\sqrt{x^2+2} - \sqrt{2} - 0}{x}$$

$$= \lim_{x \to 0} \frac{\sqrt{x^2+2} - \sqrt{2}}{x}$$

$$= \lim_{x \to 0} \frac{x^2 + \cancel{2} - \cancel{2}}{x\left(\sqrt{x^2+2} + \sqrt{2}\right)} = \frac{1}{\sqrt{2}}$$

Q_{11} $f(x) = \dfrac{x-1}{(x+1)^2}$; $x > -1$

$$f(x) = \frac{x+1-2}{(x+1)^2}$$

$$= \frac{x+1}{(x+1)^2} - \frac{2}{(x+1)^2}$$

$$= \frac{1}{x+1} - 2\,\frac{(x+1)'}{(x+1)^2}$$

$$= \frac{(x+1)'}{x+1} + 2 \cdot \frac{-(x+1)'}{(x+1)^2}$$

$$F(x) = \ln|x+1| + \frac{2}{(x+1)} + C$$

$$\to F(0) = 0$$

$$\ln|0+1| + \frac{2}{0+1} = 0$$

$$0 + 2 = 0$$

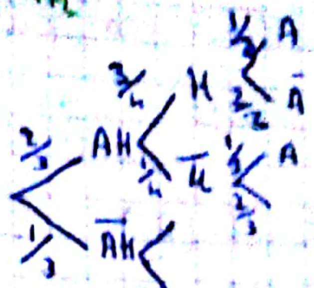
$$c = 2$$

$$F(x) = \ln(x+1) + \frac{2}{x+1} + 2$$

$$F(x) = \ln(x+1) + \frac{2 - 2x - 2}{x+1}$$

$$= \ln(x+1) - \frac{2x}{x+1} \Rightarrow (A)$$

Q_{12}



$$p(AH \cap M \cap A) = \frac{2}{3} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{6}$$

(B)

(U_n) م هندسية أساسها q

$$\Rightarrow U_{n+1} = q \cdot U_n$$

$$\Rightarrow V_n = \ln U_n \Rightarrow \ln(U_{n+1}) = \ln(q \cdot U_n)$$

$$V_{n+1} = \ln q + \ln(U_n)$$

"Concours Kech 2008"

Q_1 $z = 1 + i(3 + 5i)$

$$|z| = |1 + 3i - 5| = |-4 + 3i|$$

$$= \sqrt{(-4)^2 + (3)^2} = \sqrt{25} = 5 \Rightarrow (E)$$

Q_2 $\begin{cases} U_6 = -1 \\ U_4 = 0 \end{cases}$ $\quad$ $\mathcal{L} : P(U_n)$

$$U_n = U_p + (n-p)r$$

$$U_6 = U_4 + 2r$$

$$-1 = 0 + 2r$$

$$r = -\frac{1}{2}$$

$$U_1 = U_4 + (1-4)r$$

$$U_1 = 0 - 3\left(\frac{-1}{2}\right)$$

$$U_1 = \frac{3}{2} \Rightarrow (E)$$

Q3

$q = \frac{1}{4}$ أساسها ه.ع (U_n)

إذن $U_{n+1} = \frac{1}{4} U_n$

إذن $\vartheta_n = \ln(U_n)$ حسابية

أساسها $r = ?$

$U_{n+1} = \frac{1}{4} U_n$

$\ln(U_{n+1}) = \ln\left(\frac{1}{4}\right) + \ln(U_n)$

$\vartheta_{n+1} = -\ln 4 + \ln(U_n)$

$\vartheta_{n+1} = \underbrace{-2\ln 2}_{r} + \vartheta_n$

$r = -2\ln 2$

خاصية

ⓐ إذا كانت (U_n) م. هندسية أساسها q و $q > 0$ فإن $V_n = \ln(U_n)$ م. حسابية أساسها $\ln(q)$

ⓑ إذا كانت (U_n) م. حسابية أساسها r فإن $V_n = e^{U_n}$ هندسية أساسها $q = e^{r}$

$U_{n+1} = U_n + r \Leftrightarrow e^{U_{n+1}} = e^{U_n + r}$

$e^{U_{n+1}} = e^{U_n} \times e^{r}$

$V_{n+1} = V_n \times e^{r}$

Q4

$f(x) = \begin{cases} f(x) = a + \frac{1 - \sqrt{1 + \sin x}}{x} \\ f(0) = 0 \end{cases}$

حدد قيمة a لتكون f متصلة في 0

f متصلة في 0 $\Leftrightarrow$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} a + \frac{1 - \sqrt{1 + \sin x}}{x} = 0$

$\Leftrightarrow \lim_{x \to 0} a + \frac{1 - 1 - \sin x}{x(1 + \sqrt{1 + \sin x})} = 0$

$\Leftrightarrow \lim_{x \to 0} a - \frac{\sin x}{x} \times \frac{1}{1 + \sqrt{1 + \sin x}}$

$a - 1 \times \frac{1}{2} = 0$

Q5

$f(\ell) = \ell$

$\sqrt{\frac{2}{3}\ell^2 + 2} = \ell$

$\frac{2}{3}\ell^2 + 2 = \ell^2$

$-\frac{1}{3}\ell^2 = -2$

$\ell^2 = 6$

$\ell = \sqrt{6} \Rightarrow \boxed{B}$

Q6

$g \begin{cases} g(x) = \frac{\sqrt{1 + 2x} - 1}{x} \\ g(0) = 1 \end{cases}$

$g'(0) = ?$

$g'(0) = \lim_{x \to 0} \frac{g(x) - g(0)}{x - 0}$

$= \lim_{x \to 0} \frac{\frac{\sqrt{1 + 2x} - 1}{x} - 1}{x}$

$= \lim_{x \to 0} \frac{\sqrt{1 + 2x} - (1 + x)}{x^2}$

$= \lim_{x \to 0} \frac{\cancel{1} + \cancel{2x} - \cancel{1} - \cancel{2x} - x^2}{\cancel{x^2}(\sqrt{} + (1 + x))}$

$= \lim_{x \to 0} \frac{-1}{\sqrt{1 + 2x} + (1 + x)}$

$= -\frac{1}{2} \Rightarrow \boxed{A}$

Q_7

$h(x) = \frac{4}{4-x^2} = \frac{-4}{x^2-2^2}$

الدالة الأصلية للدالة h بحيث تأخذ 0 عند x.

$H(0) = 0$

$f(x) = \frac{k}{x^2-a^2} = \frac{k}{(x-a)(x+a)}$

$= \frac{k}{2a}\left(\frac{1}{x-a} - \frac{1}{x+a}\right)$

$f(x) = \frac{k}{2a}\left(\frac{(x-a)'}{x-a} - \frac{(x+a)'}{x+a}\right)$

$\Rightarrow F(x) = \frac{k}{2a}\left(\ln|x-a| - \ln|x+a|\right)$

$F(x) = \frac{k}{2a}\ln\left|\frac{x-a}{x+a}\right|$

$H(x) = \frac{-4}{2\times(2)}\ln\left|\frac{x-2}{x+2}\right| + C$

$C = ?$

$H(0) = \Leftrightarrow \ln(1) + C = 0$

$\Leftrightarrow \ln(1) = C$

$\Leftrightarrow C = 0$

$H(x) = \ln\left|\frac{x+2}{x-2}\right| + 0 = \ln\frac{|x+2|}{|x-2|}$

$-2 < x < 2$

$0 < x+2 < 4 \Rightarrow |x+2| = x+2$

$-4 < x-2 < 0 \Rightarrow |x-2| = 2-x$

إذن

$H(x) = \ln\left(\frac{2+x}{2-x}\right)$

Q_8

حل في $\mathbb{R}$ المعادلة

$e^{2x} - 2e^x - 3 = 0$

$\Delta = 4 + 12 = 16$

$e^x = \frac{2-4}{2} = -1 \neq$

أو

$e^x = \frac{2+4}{2} = 3 \Rightarrow x = \ln 3$

إذن B

Q_9

$\Omega = \int_a^b |f(x) - g(x)|\, dx \ \|\vec{i}\| \times \|\vec{j}\|$ u.a

مساحة الحيز المحصور بين $C_f(x)$ و $C_g(x)$ من المستوى

$x > 0$

والمستقيم $x = \frac{1}{3}$ و $x = \frac{4}{3}$

$\Omega = \int_{1/3}^{4/3} |f(x) - g(x)|\, dx$ cm^2

$f(x) > g(x) \Rightarrow f(x) - g(x) > 0$

$\Rightarrow |f(x) - g(x)| = f(x) - g(x)$

$\Omega = \int_{1/3}^{4/3} f(x) - g(x)\, dx = \int_{1/3}^{4/3} \frac{1}{x} - \frac{1}{2x}\, dx$

$\Omega = \frac{1}{2}\left[\ln x\right]_{1/3}^{4/3}$

$= \ln 2\, c^2 \Rightarrow \boxed{C}$

Q_{10}

$f(x) = 2x - \sqrt{1+x^2}$

$\lim_{x\to+\infty} \frac{f(x)}{x} = 1$

$\lim_{x\to+\infty} f(x) - x = 0$

$y = x$

Q_{11}

h دالة و $x = \frac{3}{2}$ محور تماثل

ل C_f

$$\begin{cases} x \in D_f \Rightarrow 2a - x \in D_f \\ f(2a - x) = f(x) \end{cases} \Leftrightarrow$$

المستقيم ذو المعادلة $x = a$ محور تماثل

$a = \frac{3}{2}$

$h(2a - x) = h(x)$

$h\left(2 \times \frac{3}{2} - x\right) = h(x)$

$h(3 - x) = h(x)$

Q_{12}

$\forall k \in \{1, 2, 3, 4, 5, 6\}$

$P(\{k\}) = P_k$

حدود P_1 و P_2 و P_3 و P_4 و P_5 و P_6

متتابعة لمتتالية حسابية أساسها

$r = -\frac{1}{45}$

$P_1 = ?$

$$\begin{cases} P(\Omega) = 1 \\ \Omega = \{1, 2, 3, 4, 5, 6\} \end{cases}$$

$P(\{1, 2, 3, 4, 5, 6\}) = 1$

$P(\{1\}) + P(\{2\}) + \dots + P(\{6\}) = 1$

$P_1 + P_2 + \dots P_6 = 1$

S_n مجموع عدد حدود متتابعة لمتتالية حسابية يساوي

$S_n = \frac{\text{عدد حدود } S_n}{2}$ (الحد الأخير + الحد الأول لـ S_n)

$\frac{6}{2}(P_1 + P_6) = 1$

$P_1 = P_6 - 5 \times \frac{1}{45}$

$P_1 = P_6 - \frac{1}{9}$

$\frac{6}{2}\left(P_1 + P_1 - \frac{1}{9}\right) = 1$

$6P_1 - \frac{1}{3} = 1$

$6P_1 = 1 + \frac{1}{3} = \frac{5}{3}$

$2P_1 = \frac{4}{9}$

$P_1 = \frac{2}{9} \Rightarrow \boxed{E}$

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Q_1

$\overline{z}^2 = \overline{z}^2$

$$z = \frac{1 + i\sqrt{3}}{(1 - i\sqrt{3})^2} = \frac{(1 + i\sqrt{3})(1 + i\sqrt{3})^2}{(1 - i\sqrt{3})^2(1 + i\sqrt{3})^2}$$

$$= \frac{(1 + i\sqrt{3})(-2 + 2i\sqrt{3})}{\left((1 - i\sqrt{3})(1 + i\sqrt{3})\right)^2}$$

$$= \frac{-2 - 6}{(1 + 3)^2} = \frac{-8}{16} = \frac{-1}{2}$$

$$= -\frac{1}{2} + i0$$

$Im(z) = 0 \Rightarrow C$

Q_2

$z + \frac{1}{z} = -1$

$z^2 + z + 1 = 0$

$\Delta = 1 - 4 = -3 < 0$

$z_1 = \frac{-1 + i\sqrt{3}}{2}$

$z_2 = \frac{-1 - i\sqrt{3}}{2}$

$\Rightarrow D$

Q_3:

$$f(x) = \sqrt{x^2 - 2x - 2}$$

$x \in D_f \iff x^2 - 2x - 2 \geq 0$

$\Delta = 12$

$x_1 = \frac{2 - 2\sqrt{3}}{2} = 1 - \sqrt{3}$

$x_2 = \frac{2 + 2\sqrt{3}}{2} = 1 + \sqrt{3}$

x	$-\infty$		$1-\sqrt{3}$		$1+\sqrt{3}$		$+\infty$
x^2-2x-2		$+$	0	$-$	0	$+$	

$D_f =]-\infty, 1-\sqrt{3}] \cup [1+\sqrt{3}, +\infty[$

Q_4

$$\lim_{x \to +\infty} \frac{\sqrt{2x^2+1} - \sqrt{x-1}}{x}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\left(\sqrt{2+\frac{1}{x^2}} - \sqrt{\frac{\cancel{x}}{x} - \frac{1}{x^2}}\right)}{\cancel{x}}$$

$= \sqrt{2}$

Q_5

$$\begin{cases} U_1 = 1 \\ U_{n+1} = 2U_n + \frac{n+2}{n(n+1)} \end{cases}$$

$V_n = U_n + \frac{1}{n}$

$V_{n+1} = q \cdot V_n$

$q = ?$

$V_{n+1} = 2U_n + \frac{n+2}{n(n+1)} + \frac{1}{n+1}$

$= 2U_n + \frac{n+2+\cancel{n}}{n(n+1)}$

$= 2U_n + \frac{2(n+1)}{n(n+1)} = 2\left(U_n + \frac{1}{n}\right)$

$= 2V_n$

$q = 2$

Q_6:

$$h \begin{cases} h(x) = \frac{\sin \pi x}{x-1} & x \neq 1 \\ h(1) = a \end{cases}$$

$a = ?$ بحيث تكون h مستمرة ؟

$\lim_{x\to 1} h(x) = a$

$a = \lim_{x\to1} h(x) = \lim_{x\to1} \frac{\sin \pi x}{x-1}$

$= \lim_{x\to1} \frac{\sin\pi x - \sin\pi(1)}{x-1}$

نضع $f(x) = \sin \pi x$

$f'(x) = \pi \cos \pi x \Rightarrow f'(1) = \pi\cos(1) = -\pi$

$$\lim_{x\to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0)$$

$(\sin a)' = a \cos x$

$a = \lim_{x\to1} \frac{f(x) - f(1)}{x-1} = f'(1) = -\pi$

$2: ⅁$

$a = \lim_{x\to1} \frac{\sin \pi x}{x-1}$

نضع $x = t + 1 \Rightarrow t = x - 1$

$a \to 1 \Rightarrow t \to 0$

$a = \lim_{t\to0} \frac{\sin\pi(t+1)}{t}$

$= \lim_{t=0} \frac{\sin(\pi t + \pi)}{t}$

$= \lim_{t\to0} \frac{-\sin\pi t}{t} = -t$

Q_7

$\forall x \in]0, +\infty[$

$g(x) = x\, g\left(\frac{1}{x}\right)$ et $g(1) = 1$

$[g(f(x))]' = f'(x)\, g'(f(x))$

$g(x) = x\, g\left(\frac{1}{x}\right)$

$g'(x) = \left(x\, g\left(\frac{1}{x}\right)\right)'$

$g'(x) = (x)'\left(g\left(\frac{1}{x}\right)\right) + x\left(g\left(\frac{1}{x}\right)\right)'$

$= g\left(\frac{1}{x}\right) + x\left(\frac{1}{x}\right)' g'\left(\frac{1}{x}\right)$

$= g\left(\frac{1}{x}\right) + x\left(-\frac{1}{x^2}\right) g'\left(\frac{1}{x}\right)$

$= g'\left(\frac{1}{x}\right) - \frac{1}{x}\, g'\left(\frac{1}{x}\right)$

$x = 1 \in]0, +\infty[$

$\Rightarrow g'(1) = g(1) - \frac{1}{1} g'(1)$

$\Rightarrow 2g'(1) = g(1)$

$\Rightarrow g'(1) = \frac{g(1)}{2} = \frac{1}{2}$

$\Rightarrow$ C

Q9

$\int_0^2 \frac{|1-x|}{|1-x^2| + |1+x^2|}\, dx$

$\int_a^b f(x)\, dx \Rightarrow x \in [a,b]$

$|x| \begin{cases} x & \text{si } x > 0 \\ -x & \text{si } x \leq 0 \end{cases}$

x	0		1		$+\infty$
$1-x$		$+$	0	$-$	
$1-x^2$		$+$	0	$-$	

$|1-x| \rightarrow 1-x \quad x \in]0,1[$

$\rightarrow x-1 \quad x \in]1, +\infty[$

$\int_0^2 \frac{|1-x|}{|1-x^2| + 1 + x^2}\, dx$

$= \int_0^1 \frac{1-x}{1-\cancel{x^2}+1+\cancel{x^2}}\, dx + \int_1^2 \frac{x-1}{x^2 \cancel{-1} \cancel{+1} + x^2}\, dx$

$= \frac{1}{2}\int_0^1 1 - x\, dx + \int_1^2 \frac{x-1}{2x^2}\, dx$

$= \frac{1}{2}\left[x - \frac{1}{2}x^2\right]_0^1 + \frac{1}{2}\int_1^2 \frac{1}{x} - \frac{1}{x^2}\, dx$

$= \frac{1}{2}\left[1 - \frac{1}{2} - 0\right] + \frac{1}{2}\left[\ln x + \frac{1}{x}\right]_1^2$

$= \frac{1}{4} + \frac{1}{2}\left[\ln 2 + \frac{1}{2} - 1\right]$

$= \cancel{\frac{1}{4}} + \frac{1}{2}\ln 2 - \cancel{\frac{1}{4}} = \frac{\ln 2}{2}$

$\Rightarrow$ D

Q9 :

$f(x) = x + \frac{x}{\sqrt{1+2x^2}}$

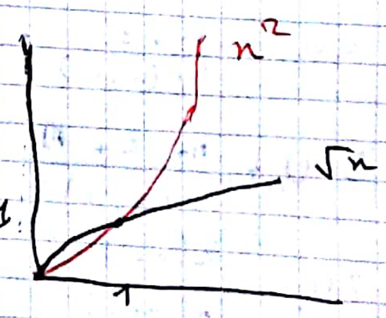
مقارب مائل Q8

$\lim_{x \to +\infty} \frac{f(x)}{x} = 1$

$\lim_{x \to +\infty} f(x) - x = \frac{1}{\sqrt{2}}$

$y = x + \frac{1}{\sqrt{2}}$ $\Rightarrow$ E

Q10



$\mathcal{A} = \int_0^1 \sqrt{x} - x^2 \, dx + \int_1^2 x^2 - \sqrt{x} \, dx$

$= \left[\frac{2}{3} x\sqrt{x} - \frac{1}{3} x^3\right] + \left[\frac{1}{3}x^3 - \frac{2}{3} x\sqrt{x}\right]_1^2$

$= 2\left(\frac{5 - 2\sqrt{2}}{3}\right) cm^2$

السؤال 11 :

$\begin{cases} x \in D_f \Rightarrow 2a - x \in D_f \\ f(2a - x) + f(x) = 2b \end{cases}$

$\Omega(1,2)$ مركز تماثل لـ f

$f((2\times 1) - x) + f(x) = 4$

$h(2 - x) + h(x) = 4$

Q_{12} نرمي في آن واحد

D_1 \ D_2	1	2	3	4	5	6
1	1,1	(2,1)	3,1			
2						2,6
3					3,5	
4				4,4		
5			5,3			
6		6,2				

$p(2)\times p(6) + p(6)\times p(2) + p(4)\times p(4) + p(3)\times p(5)$
$+ p(5)\times p(3) = \frac{5}{36}$

Cncrs Kech 2010

Q_1 $\ln 3 + 4\ln 2 - \ln 60$

$\ln 3 + 2\ln 4 - \ln 5\times 4\times 3$

$\cancel{\ln 3} + 2\ln 4 - \ln 5 - \ln 4 - \cancel{\ln 3}$

$\ln 4 - \ln 5 = \ln\left(\frac{4}{5}\right) \Rightarrow (E)$

Q_2

$\mathrm{Im}\left(z = \frac{1+ix}{1-ix}\right) = ?$

$z = \frac{1+ix}{1-ix} = \frac{(1+ix)^2}{1+x^2} = \frac{1-x^2+2ix}{1+x^2}$

$= \frac{1-x^2}{1+x^2} + i\boxed{\frac{2x}{1+x^2}} \Rightarrow (D)$

Q_3

$\left(\frac{1}{13}\right)^{x^2-3x} = 13^2$

$(13)^{-(x^2-3x)} = 13^2$

$\Leftrightarrow -x^2 + 3x = 2$

$\Delta = 9 - 8 = 1$ $\begin{cases} x_1 = \frac{3-1}{2} = 1 \\ x_2 = \frac{3+1}{2} = 2 \end{cases}$

$S = \{1, 2\} \Rightarrow (C)$

Q_4

العدد j

$j = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

لدينا

$j^3 = 1$

$j^{3k} = (j^3)^k = 1$

$1 + j + j^2 = 0$

$j = \left[1, \frac{2\pi}{3}\right]$

$$1 + j + j^2 + \cdots j^{2010} = \frac{j^{2012} - 1}{j - 1}$$

$$S = \frac{j^{670 \times 3} \times j - 1}{j - 1} = \frac{j - 1}{j - 1} = 1 \Rightarrow \boxed{A}$$

Q_5 :
$$u = \sqrt[3]{\frac{2}{7}}$$
$$u_{n+1} = \sqrt{1}$$

Q_8

$f \circ f$ فردية f

$f(-x) = -f(x)$

$$f \circ f(-x) = f(f(-x)) = f(-f(x))$$
$$= -f(f(x))$$
$$= -f \circ f(x)$$

اذن f فردية

زوجية + فردية ⇐ زوجية فردية + فردية ⇐ فردية
زوجية + زوجية ⇐ زوجية

Q_9

$$\lim_{x \to 0} \frac{4^x - 2^x}{x}$$
$$= \lim_{x \to 0} \frac{e^{x \ln 4} - e^{x \ln 2}}{x}$$
$$= \lim_{x \to 0} \frac{e^{2x \ln 2} - e^{x \ln 2}}{x}$$
$$= \lim_{x \to 0} e^{x \ln 2} \left(\frac{e^{x \ln 2} - 1}{x \ln 2} \right) \times \ln 2$$
$$= 1 \times 1 \times \ln 2 = \ln 2$$

$$\lim_{0} \frac{e^x - 1}{x} = 1$$

Q_{10}

$$g(x) = h\left(\cos\left(\frac{\pi}{2} x\right)\right)$$
$$g'(1) = ?$$

$$(\cos ax)' = -a \sin x$$
$$(f \circ g)' = g'(x) f'(g(x))$$

$$g'(x) = \left(h \circ \cos\left(\frac{\pi}{2} x\right)\right)' = \left(h\left(\cos\left(\frac{\pi}{2} x\right)\right)\right)'$$
$$g'(x) = \left(\cos\left(\frac{\pi}{2} x\right)\right)' \cdot h'\left(\cos \frac{\pi}{2} x\right)$$
$$g'(x) = -\frac{\pi}{2} \sin\left(\frac{\pi}{2} x\right) h'\left(\cos \frac{\pi}{2} x\right)$$
$$g'(1) = -\frac{\pi}{2} \sin \frac{\pi}{2} h'\left(\cos \frac{\pi}{2}\right)$$
$$g'(1) = -\frac{\pi}{2} h'(0) \Rightarrow A$$

Q_{11}

$$f(x) = \frac{ax + b}{cx + d}$$

$I\left(\frac{-d}{c}, \frac{a}{c}\right)$ * مركز تناظر ل C_f

$f(x) = ax^2 + bx + c \Rightarrow$ C_f محور تناظر ل $x = \frac{-b}{2a}$

$f(x) = \ln(ax^2 + bx + c) \Rightarrow$ C_f محور تناظر ل $x = \frac{-b}{2a}$

$$f(x) = \frac{5x + 1}{-2x + 1}$$

$$\Omega\left(\frac{+1}{2}, \frac{-5}{2}\right)$$

السؤال 12:

$$p(1,1,3) + p(2,2,2) + p(3,1,1)$$
$$+ p(2,2,1) + p(2,2,1) + p(2,2,1)$$

B	V	R
1	1	3
1	3	1
3	1	1
2	2	1
2	1	2
1	2	2

$$\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}+$$
$$\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}=\frac{6}{216}=\frac{1}{36}$$

Cncrs Marrakech 2011

Q21 $f(x)=\sqrt{\ln(x^2+3x-4)}$

$x\in D_f \iff x^2+3x-4>0$ و $\ln(x^2+3x-4)\geq 0$

$\iff x^2+3x-4>0$ و $x^2+3x-4\geq 1$

$\iff x^2+3x-4\geq 1$

$x>a$ و $x>b \Rightarrow x>\sup(a,b)$

$x<a$ و $x<b \Rightarrow x<\inf(a,b)$

$\iff x^2+3x-4=1$

$\iff x^2+3x-5=0$

$\Delta=9+5\times 4=29$ $\left\langle \begin{array}{l} x_1=\dfrac{-3-\sqrt{29}}{2} \\ x_2=\dfrac{-3+\sqrt{29}}{2}\end{array}\right.$

x	$-\infty$		$\frac{-3-\sqrt{29}}{2}$		$\frac{-3+\sqrt{29}}{2}$	
x^2+3x-5		$+$	0	$-$	0	$+$

$D_f=\left]-\infty,\frac{-3-\sqrt{29}}{2}\right]\cup\left[\frac{-3+\sqrt{29}}{2},+\infty\right[$ C

Q22

$$\lim_{x\to+\infty}\frac{x-\sqrt{x^2+1}}{x+\sqrt{x^2-1}}$$

$$=\lim_{x\to+\infty}\frac{x\left(1-\sqrt{1+\frac{1}{x^2}}\right)}{x\left(1+\sqrt{1-\frac{1}{x^2}}\right)}=\frac{1-1}{1+1}=\frac{0}{2}=C$$

Q23

$$\begin{cases} g(x)=\dfrac{\operatorname{tg}x-\sin x}{x} & x\neq 0\\ g(0)=u\end{cases}$$

0 في متصلة g تكون $u=?$

$g(0)=\lim_{x\to 0}g(x)$

$$\lim_{x\to 0}\frac{\operatorname{tg}x-\sin x}{x^3}$$

$$=\lim_{x\to 0}\frac{\operatorname{tg}x-\cos x\operatorname{tg}x}{x^3}$$

$$=\lim_{x\to 0}\frac{\operatorname{tg}x(1-\cos x)}{x^3}$$

$$=\lim_{x\to 0}\frac{\operatorname{tg}x}{x}\times\frac{(1-\cos x)}{x^2}$$

$$=\frac{1}{2}\Rightarrow D$$

Q24

z^2+2z-3 / $z=x+iy$

$x=?$ $y=?$ / $z^2+2z-3\in\mathbb{R}$

$z^2+2z-3\in\mathbb{R}\iff Im=0$

$z^2+2z-3=(x+iy)^2+2(x+iy)-3$

$=x^2-y^2+2iyx+2x+2iy-3$

$z^2+2z-3=(x^2-y^2+2x-3)+i(2xy+2y)$

$Im=0\iff 2xy+2y=0$

$2y(x+1)=0$

C $\Leftarrow y=0$ و $x=-1$

Q25 :

$$S_n = U_3 + U_4 + U_5 + \dots + U_{10} = 672$$

$$\Leftrightarrow \frac{10-3+1}{2}(U_3 + U_{10}) = 672$$

$$U_3 + U_{10} = 168$$

$$U_7 + (3-7)r + U_7 + (10-7)r = 168$$

$$2U_7 - 4r + 3r = 168$$

$$2U_7 - r = 168$$

Q26

$$S = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \dots + \frac{1}{512}$$

S_n : الحد الأول $\left(\dfrac{1 - (\text{الأساس})^{\text{عدد الحدود}}}{1 - \text{الأساس}} \right)$

$$S_n = \frac{1}{2}\left(1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \dots + \left(-\frac{1}{2}\right)^8\right)$$

$$1 + b + b^2 + \dots + b^n = \frac{b^n - 1}{b - 1}$$

$$S_n = \frac{1}{2}\left[\frac{1 - \left(-\frac{1}{2}\right)^9}{1 + \frac{1}{2}}\right]$$

$$= \frac{1}{2}\left[\frac{1 + \left(\frac{1}{2}\right)^9}{\frac{3}{2}}\right]$$

$$= \frac{1}{3}\left[1 + \left(\frac{1}{2}\right)^9\right] = \frac{171}{512}$$

Q27

$$I = \int_{-1}^{+1} \frac{1}{x^2 - 4}$$

$$I \int \frac{1}{ax^2 + bx + c}$$

$(\Delta > 0)$

x_1, x_2 solut

$$I = \int \frac{\alpha}{x - x_1} + \frac{\beta}{x - x_2}$$

$$= \int \alpha \frac{(x - x_1)'}{x - x_1} + \beta \frac{(x - x_2)'}{x - x_2}$$

06

$$= \alpha \ln|x - x_1| + \beta \ln|x - x_2|$$

$$= \ln|x - x_1|^\alpha + \ln|x - x_2|^\beta$$

$$= \ln\left(|x - x_1|^\alpha \times |x - x_2|^\beta\right)$$

$$I = \int_{-1}^{1} \frac{1}{x^2 - 4}\, dx$$

$$= \frac{1}{4}\int_{-1}^{1} \frac{1}{x-2} - \frac{1}{x+2}\, dx$$

$$= \frac{1}{4}\left[\ln|x-2| - \ln|x+2|\right]$$

$$= \frac{1}{4}\left[\ln\left|\frac{x-2}{x+2}\right|\right]_{-1}^{1}$$

$$= \frac{1}{4}\left[\ln\left|\frac{x-2}{x+2}\right|\right]_{-1}^{1}$$

$$= \frac{1}{4}\left(\ln\frac{1}{3} - \ln 3\right]$$

$$= -\frac{1}{2}\ln 3 .$$

Q29

$\int_a^b f(x)\, dx = F(b) - F(a)$ تكامل

$\int f(x)\, dx = F(x) + c$ دالة أصلية

$$f(x) \frac{\ln x}{x^3}$$

$\int \frac{1}{x^3} \ln x \, dx$ I.P.P

نضع $\begin{cases} u(x) = \ln x \\ v'(x) = \frac{1}{x^3} = x^{-3} \end{cases} \Leftarrow \begin{cases} u'(x) = \frac{1}{x} \\ v(x) = \frac{1}{-3+1} x^{-3+1} \end{cases}$

$$\int \frac{1}{x^3} \ln x \, dx = \left[\frac{-1}{2x^2} \ln x\right] + \frac{1}{2}\int \frac{1}{x^3}\, dx$$

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$I = \frac{-1}{2x^2}\ln(x) + \frac{1}{2}\left[\frac{-1}{2x^2}\right] + C$

$F(x) = -\frac{1}{2x^2}\ln x - \frac{1}{4x^2} + C$

$F(1) = 0$

$\Leftrightarrow -\frac{1}{2(1)}\ln(1) - \frac{1}{4(1)^2} + C = 0$

$C = \frac{1}{4}$

$F(x) = -\frac{1}{2x^2}\ln x - \frac{1}{4x^2} + \frac{1}{4} \Rightarrow (B)$

Q29

معادلة المماس ل f في النقطة ذات الأفصول x_0 هي

$y = f'(x_0)(x - x_0) + f(x_0)$

$(f(g(x))' = g'(x)\, f'(g(x))$

$y = f'(0)(x-0) + f(0)$

$f'(x) = (\cos(e^x))' = (e^x)'\cos'(e^x)$

$f'(x) = -e^x \sin(e^x)$

$\Rightarrow f'(0) = -e^0 \sin e^0$

$f'(0) = -1\sin(1)$

d'où : $y = -\sin(1)(x-0) + \cos(e^0)$

$y = -\sin(1)x + \cos 1 \Rightarrow (C)$

Q30

$z = \frac{\sqrt{3}+i}{\sqrt{2}-i\sqrt{2}}$

$= \frac{[2, \frac{\pi}{6}]}{[2, -\frac{\pi}{4}]} = \left[\frac{2}{2}, \frac{\pi}{6} - \left(-\frac{\pi}{4}\right)\right]$

$= \left[1, \frac{5\pi}{12}\right]$

$\arg(z) \equiv \frac{5\pi}{12} \Rightarrow (C)$

Cncrs Kech 2012

Q21

(U_n) حسابية

$\begin{cases} U_6 = 25 \\ U_2 + U_3 + U_4 = 21 \end{cases}$

$\boxed{\begin{array}{ll} U_n = U_p + (n-p)r & (U_n) \text{ ح} \\ U_n = q^{n-p} U_p & (U_n) \text{ ه} \end{array}}$

$\begin{cases} U_6 = 25 \\ U_0 + 2r + U_0 + 3r + U_0 + 4r = 21 \end{cases}$

$\begin{cases} U_6 = 25 = U_0 + 6r \\ 3U_0 + 9r = 21 \end{cases}$

$\begin{cases} ① & U_0 + 6r = 25 \\ ② & U_0 + 3r = 7 \end{cases}$

$① - ② \Leftrightarrow 3r = 18$

$\Leftrightarrow r = 6$

$① \Rightarrow U_0 + 6r = 25$

$\Rightarrow U_0 + 36 = 25$

$\Rightarrow U_0 = -11 \Rightarrow (C)$

Q22

$\lim_{x\to+\infty} \sqrt{x^2+x+1} - \sqrt{x^2-x+1} + (x^2)^{1/x}$

$a^x = e^{x\ln(a)}$

$\lim_{x\to+\infty} \frac{\sqrt{x^2+x+1}^2 - \sqrt{x^2-x+1}^2}{\sqrt{x^2+x+1} + \sqrt{x^2-x+1}} + (x^2)^{1/x}$

$\lim_{x\to+\infty} \frac{x^2+x+1 - x^2+2x-1}{x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + \sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)} + e^{2\frac{\ln x}{x}}$

$= \lim_{x\to+\infty} \frac{2\cancel{x}}{\cancel{x}\left(\sqrt{\quad} + \sqrt{\quad}\right)} + e^{2\frac{\ln x}{x}}$

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$= 2 \Rightarrow \boxed{A}$

Q23

ط 1 ب

$$h \begin{cases} h(x) = \dfrac{\sin\left(2x + \frac{\pi}{3}\right)}{x - \frac{\pi}{3}} & x \neq \frac{\pi}{3} \\ h\left(\frac{\pi}{3}\right) = a \end{cases}$$

$a = ?$ بحيث f متصلة في $\frac{\pi}{3}$

$$h\left(\frac{\pi}{3}\right) = \lim_{x \to \frac{\pi}{3}} h(x)$$

$$a = \lim_{x \to \frac{\pi}{3}} \frac{\sin\left(2x + \frac{\pi}{3}\right)}{x - \frac{\pi}{3}}$$

ط 1 ب

$$t = x - \frac{\pi}{3}$$

$$x \to \frac{\pi}{3} \Rightarrow t \to 0$$

$$a = \lim_{t \to 0} \frac{\sin\left(2\left(t + \frac{\pi}{3}\right) + \frac{\pi}{3}\right)}{t}$$

$$= \lim_{t \to 0} \frac{\sin(2t + \pi)}{t} = \lim_{t \to 0} \frac{-\sin 2t}{t}$$

$$= \lim_{t \to 0} -\frac{2 \sin 2t}{2t} = -2$$

ط 2 ب

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$(\cos(ax + b))' = -a \sin(ax + b)$$

$$(\sin(ax + b))' = a \cos(ax + b)$$

$$a = \lim_{x \to \frac{\pi}{3}} \frac{\sin\left(2x + \frac{\pi}{3}\right) - 0}{x - \frac{\pi}{3}}$$

$$= \lim_{x \to \frac{\pi}{3}} \frac{f(x) - f\left(\frac{\pi}{3}\right)}{x - \frac{\pi}{3}} = f'\left(\frac{\pi}{3}\right)$$

$$f(x) = \sin\left(2x + \frac{\pi}{3}\right) \Rightarrow f\left(\frac{\pi}{3}\right) = \sin\left(\frac{2\pi}{3} + \frac{\pi}{3}\right)$$

$$= \sin \pi = 0$$

$$f'(x) =$$

$$f'(x) = 2\cos\left(2x + \frac{\pi}{3}\right)$$

$$f'\left(\frac{\pi}{3}\right) = 2\cos\left(\frac{2\pi}{3} + \frac{\pi}{3}\right) = 2\cos \pi$$

$$= 2 \cdot (-1)$$

$$= -2 \Rightarrow D$$

Q24

$$f(x) = \ln\left(5 - |x - 1| - |5x - 1|\right)$$

D_f : ?

$$x \in D_f \Leftrightarrow 5 - |x - 1| - |5x - 1| > 0$$

x	$-\infty$ $\qquad$ $\frac{1}{5}$	$\qquad$ 1	
$\lvert 5x - 1\rvert$	$-5x + 1$	$5x - 1$	$5x - 1$
$\lvert x - 1\rvert$	$1 - x$	$1 - x$	$x - 1$
$5 - \lvert x - 1\rvert - \lvert 5x - 1\rvert$	$5 - 1 + x + 5x - 1$ $= 6x + 3$	$5 + x - 1 - 5x - 1$ $=$	

إذا كان $x \in \left]-\infty, \frac{1}{5}\right]$

$$D_f \Leftrightarrow 6x + 3 > 0$$

$$\Leftrightarrow x > -\frac{1}{2} \text{ , } x \leq \frac{1}{5}$$

$$\Leftrightarrow -\frac{1}{2} < x \leq \frac{1}{5} \Leftrightarrow x \in \left]-\frac{1}{2}, \frac{1}{5}\right]$$

Q25

$$f(x) = 1 + 2x + 3x^2 + \dots + 100x^{99}$$

$$f(x) = (x)' + (x^2)' + (x^3)' + \dots + (x^{100})'$$

$$f(x) = (x + x^2 + x^3 + \dots + x^{100})'$$

$$b + b^2 + b^3 + \dots + b^n = b\left(\frac{1 - b^n}{1 - b}\right)$$

$$f(x) = \left(x\left(\frac{1 - x^{100}}{1 - x}\right)\right)' = \left(\frac{x^{101} - x}{x - 1}\right)'$$

$$f(x) = \frac{(x^{101} - x)'(x - 1) - (x^{101} - x)(x - 1)'}{(x - 1)^2}$$

$$f(-1) = \frac{(101-1)(-1-1)-(-1+1)}{(-1-1)^2}$$

$$= \frac{200}{4} = 50 \rightarrow D$$

Q26

$$= \int_0^1 \frac{1}{x^2-x-1}$$

$$\Delta = (-1)^2 + 4 = 5 \begin{cases} x_1 = \frac{1-\sqrt{5}}{2} \\ x_2 = \frac{1+\sqrt{5}}{2} \end{cases}$$

$$\frac{1}{x^2-x-1} = \frac{+1}{\sqrt{5}}\left[\frac{1}{x-\left(\frac{1-\sqrt{5}}{2}\right)} - \frac{1}{x-\left(\frac{1+\sqrt{5}}{2}\right)}\right]$$

$$= \frac{\left(x-\left(\frac{-1+\sqrt{5}}{2}\right)-x-\frac{-1-\sqrt{5}}{2}\right)\sqrt{5}}{x^2-x-1}$$ Bgaoubot

$$\int \frac{1}{x^2-x-1}\,dx = \frac{+1}{\sqrt{5}}\left[\ln\left|x-\left(\frac{1-\sqrt{5}}{2}\right)\right| - \ln\left|x-\left(\frac{1+\sqrt{5}}{2}\right)\right|\right]_0^1$$

2 D

$$I = \frac{+1}{\sqrt{5}}\ln\left|\frac{x-\left(\frac{1-\sqrt{5}}{2}\right)}{x-\left(\frac{1+\sqrt{5}}{2}\right)}\right|\Bigg]_0^1$$

$$= \frac{+1}{\sqrt{5}}\left[\ln\left(\frac{\sqrt{5}+1}{\sqrt{5}-1}\right) - \ln\left(\frac{\sqrt{5}+1}{\sqrt{5}-1}\right)\right.$$

$$= +\frac{2}{\sqrt{5}}\ln\left(\frac{(\sqrt{5}+1)^2}{5-1}\right)$$

$$= \frac{+2}{\sqrt{5}}\ln\left(\frac{1+2\sqrt{5}+5}{4}\right) = \frac{+2}{\sqrt{5}}\ln\left(\frac{6+2\sqrt{5}}{4}\right)$$

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tmars

Q27

$P(z)= z^3+(\sqrt{3}-i)z^2+(1-i\sqrt{3})z-i$ ①

عند حل الأول ... أو ...

⇐ أو ... حل ل ...

$S=\{4 ...\}$ $S=\{-i, ...$

$P(z)=(z-i)(z^2+bz+c)$

$P(z)=z^3+(b-i)z^2+(c-ib)z-ic$

$P(z)=z^3+(b-i)z^2+(c-ib)z-ic$ ②

من ① و ②

$$\begin{cases} b-i=\sqrt{3}-i \Rightarrow b=\sqrt{3} \\ -ic=-i \Rightarrow c=1 \end{cases}$$

$P(z)=0 \iff (z-i)(z^2+\sqrt{3}z+1)=0$

$z=i$ أو $z^2+\sqrt{3}z+1=0$

$\Delta=(\sqrt{3})^2-4(1)=-1$

$\Delta=(i)^2$

$z_1=\frac{-\sqrt{3}-i}{2}$ و $z_2=\frac{-\sqrt{3}+i}{2} \Rightarrow$ A

Q28

صيغ المرور من الداء إلى الجمع

$\cos a\cos b=\frac{1}{2}[\cos(a+b)+\cos(a-b)]$

$\sin a\cos b=\frac{1}{2}[\sin(a+b)+\sin(a-b)]$

$\sin a\sin b=\frac{1}{2}[\cos(a-b)-\cos(a+b)]$

ط 1:

$\int\cos x\cos 2x=\frac{1}{2}\int\cos(x+2x)+\cos(x-2x)\,dx$

$=\frac{1}{2}\int\cos 3x+\cos x\,dx$

$=\frac{1}{2}\left[\frac{1}{3}\sin 3x+\sin x\right]$

$F(x)=\frac{1}{6}\sin 3x+\frac{1}{2}\sin x+C$

$F(0)=0 \Rightarrow \frac{1}{2}\sin(0)+\frac{1}{2}\sin(0)+C=0$

$C=0$

$F(x)=\frac{1}{6}\sin 3x+\frac{1}{2}\sin x$

$\cos 2x=2\cos^2x-1$

$\cos 2x=1-2\sin^2x$

$\sin 2x=2\cos x\sin x$

ط 2:

$\int\cos x\cos 2x\,dx=\int\cos x(1-2\sin^2(x))\,dx$

$\int\cos x-2\cos x(\sin x)^2\,dx=\int\cos x-(\sin x)'\sin ...$

$=\sin x-2\cdot\frac{1}{2+1}(\sin x)^{2+1}+C$

$F(x)=\sin x-\frac{2}{3}\sin^3x+C$

$F(0)=0 \Rightarrow \underbrace{\sin 0}_{0}-\frac{2}{3}\underbrace{\sin^3 0}_{0}+C=0$

$C=0$

$F(x)=\sin x-\frac{2}{3}\sin^3 x \Rightarrow$ C

Q29

$f(x)=\frac{1+\ln x}{x}$

معادلة المماس ل C_f في النقطة ذات الأفصول $e^{-1/2}$

معادلة المماس ل C_f في النقطة ذات الأفصول x_0 هي

$y=f'(x_0)(x-x_0)+f(x_0)$

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$y = f'(e^{-1/2})(x - e^{-1/2}) + f(e^{-1/2})$

$f'(x) = \left(\frac{1+\ln x}{x}\right)' = \frac{(1+\ln x)' x - (1+\ln x) x'}{x^2}$

$f'(x) = \frac{\frac{1}{x} x - 1 - \ln x}{x^2} = \frac{-\ln x}{x^2}$

$f'(e^{-1/2}) = -\frac{\ln e^{-1/2}}{(e^{-1/2})^2} = \frac{\frac{1}{2}}{e^{-1}} = \frac{e}{2}$

$f(e^{-1/2}) = \frac{1 + \ln(e^{-1/2})}{e^{-1/2}} = \frac{1 - \frac{1}{2}}{e^{-1/2}}$

$f(e^{-1/2}) = \frac{1}{2e^{-1/2}}$

$y = \frac{e}{2}(x - e^{-1/2}) + \frac{1}{2e^{-1/2}}$

$y = \frac{e}{2}x - \frac{e^{1/2}}{2} + \frac{e^{1/2}}{2}$

$y = \frac{e}{2}x$

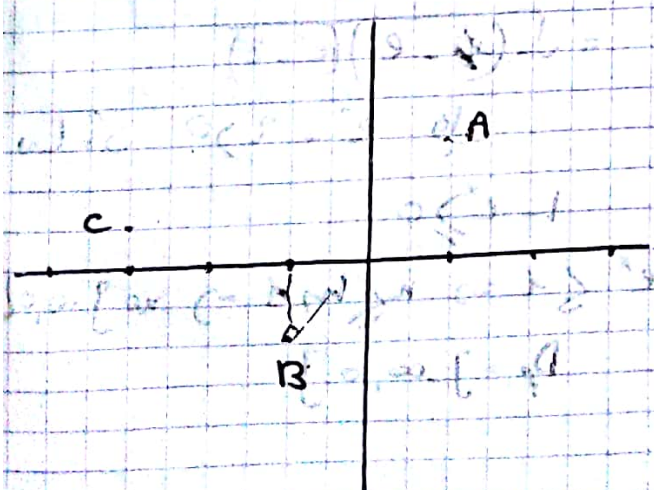
Q30

$\frac{a+ib}{-b+ia} = \frac{-i(ia-b)}{-b+ia}$

$z_A = 1 + i\sqrt{3}$

$z_B = -1 - i$

$z_C = -(2+\sqrt{3}) + i$



$\frac{z_A - z_C}{z_C - z_B} = \frac{1 + i\sqrt{3} + 1 - i}{-2 - \sqrt{3} + i + 1 + i}$

$= \frac{2 + i(\sqrt{3}+1)}{-(1+\sqrt{3}) + 2i}$

$= \frac{-i(2i - (\sqrt{3}+1))}{(-(1+\sqrt{3}) + 2i)}$

$= -i = [1, -\frac{\pi}{2}]$

قائم الزاوية في B

(B)

$\boxed{\text{Concrs } 2013}$

Q21

$\sqrt{2} - 2 + 2\sqrt{2} \dots - 64 + 64\sqrt{2} - 128$

$= \sqrt{2}\left(1 + (-\sqrt{2})^1 + (-\sqrt{2})^2 \dots + (-\sqrt{2})^{13}\right)$

$1 + b^1 + b^2 + \dots + b^n = \frac{b^{n+1} - 1}{b - 1}$

$= \sqrt{2}\left[\frac{(-\sqrt{2})^{13+1} - 1}{-\sqrt{2} - 1}\right]$

$= \sqrt{2}\left(\frac{128 - 1}{-\sqrt{2} - 1}\right) = \frac{-\sqrt{2}(127)}{\sqrt{2} + 1}$

$= \frac{-127\sqrt{2}}{\sqrt{2} + 1} \Rightarrow$ (D)

Q22 $\lim_{+\infty} \frac{\sin 2n}{3n} + \left(1 + \frac{4}{n}\right)^n$

$\boxed{a^x = e^{x \ln a}}$

$\lim_{n \to 0} \frac{\sin x}{x} = 1$

$\sin(\infty) \Rightarrow -1 \leq \sin(n) \leq 1$

$h \in [-1, 1]$

$\sin 2n = K$

gedarme ونحصر

$$\lim_{n\to+\infty} \frac{\sin(2n)}{3n} + e^{n\ln\left(1+\frac{4}{n}\right)}$$

$$\lim_{n\to+\infty} \frac{h}{3n} + e^{\frac{\ln\left(1+\frac{4}{n}\right)}{\frac{4}{n}}\times 4}$$

$$= 0 + e^{1\times 4} = e^4$$

Q_{23}

$$f(x) = \frac{x+x^2+x^3+\dots+x^9-9}{(2-x)^9-1}$$

حدد قيمة λ بحيث تكون f متصلة في 1

f متصلة $\Rightarrow \lim_{x\to 1} f(x) = f(1)$

$$\Rightarrow f(1) = \lim_{x\to 1} f(x)$$

$$\Rightarrow \lambda = \lim_{x\to 1} \frac{x+x^2+x^3+\dots+x^9-9}{(2-x)^9-1}$$

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$$

$$x^n-1 = (x-1)\left(x^{n-1}+x^{n-2}+x^{n-3}+\dots+x+1\right)$$

$$x^5-1 = (x-1)\left(x^4+x^3+x^2+x+1\right)$$

$$1+2+3+\dots+n = \frac{n}{2}(1+n)$$

$$= \lim_{x\to 1} \frac{(x-1)(\quad)}{(x-1)(\quad)}$$

$$\lambda = \lim_{1} \frac{(x-1)+(x^2-1)+(x^3-1)+\dots+x^9-9}{((2-x)-1)\left[(2-x)^8+(2-x)^7+\dots+(2-x)+1\right]}$$

$$\lambda = \lim_{1} \frac{(x-1)\left[1+(x+1)+(x^2+x+1)+\dots+(x^8+x^7+\dots+x+1)\right]}{(1-x)\left[(2-x)^8+(2-x)^7+\dots+(2-x)+1\right]}$$

$$\lambda = -\frac{1+2+3+\dots+9}{\underbrace{1+1+\dots+1+1}_{9 \text{ مرات}}} = -5 \quad C$$

$$= -\frac{\frac{9}{2}(1+9)}{9}$$

$$\lim_{x\to 1} \frac{x+x^2+x^3+\dots+x^9-9}{(2-x)^9-1}$$

$$f(x) = x+x^2+x^3+\dots+x^9-9$$

$$g(x) = (2-x)^9-1$$

$$f'(x) = 1+2x+3x^2+\dots+9x^8$$

$$g'(x) = 9(2-x)'(2-x)^{9-1} = -9(2-x)^8$$

$$f'(1) = 1+2+3+\dots+9$$

$$g'(1) = -9(2-1)^8 = -9$$

$$\lim_{x\to 1} \frac{x+x^2+\dots+x^9-9}{(2-x)^9-1} = \frac{f'(1)}{g'(1)} = \frac{1+9\dots}{-9}$$

$$= \frac{45}{-9} = -5$$

Q_{24}

D_f $\quad f(x) = \sqrt{-e^{2x}-e^x+2}$

$$x\in D_f \iff -e^{2x}-e^x+2 \geq 0$$

$$\Delta = (-1)^2+8 = 9$$

$$e^x = 1$$

$$e^x = -2 \quad \text{لا تقبل الحل}$$

$$f(x) = \sqrt{-(e^x-1)(e^x+2)}$$

$$= \sqrt{(1-e^x)(e^x+2)}$$

بما أن $e^x+2>0$ فإن

$$1-e^x \geq 0$$

$$e^x \leq 1 \Rightarrow x \leq \ln 1 \Rightarrow x\in\left]-\infty,0\right]$$

$$D_f = \left]-\infty,0\right]$$

Q25 $g(x) = \frac{\sin^3 x}{(1+\cos x)^2}$

دالة أصلية لـ g :

$\sin^2 x = 1 - \cos^2 x$

$= (1-\cos x)(1+\cos x)$

$(1+\cos x)' = -\sin x$

$f(x)$	$F(x)$
$\frac{u'(x)}{u(x)}$	$\ln\lvert u(x)\rvert$

$g(x) = \frac{\sin x \sin^2 x}{(1+\cos x)^2} = \frac{\sin x(1-\cos^2 x)}{(1+\cos x)^2}$

$= \frac{\sin x(1-\cos x)}{1+\cos x} = \frac{\sin x - \sin x\cos x}{1+\cos x}$

$= \frac{\sin x}{1+\cos x} - \frac{\sin x \cos x}{1+\cos x}$

$= -\frac{-\sin x}{1+\cos x} - \frac{\sin x\cos x + \sin x - \sin x}{1+\cos x}$

$g(x) = -\frac{(1+\cos x)'}{(1+\cos x)} - \frac{\sin x(\cos x+1)}{1+\cos x} + \frac{-\sin x}{1+\cos x}$

$= -\frac{(1+\cos x)'}{(1+\cos x)} - \sin x - \frac{(1+\cos(x))'}{1+\cos x}$

$G(x) = -\ln\lvert 1+\cos x\rvert + \cos x - \ln\lvert 1+\cos x\rvert$

$= -2\ln\lvert 1+\cos x\rvert + \cos x$

$\Rightarrow$ B

Q26

$\int_a^b f(x)\,dx + \int_a^b g(x)\,dx = \int_a^b [f(x)+g(x)]\,dx$

$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x}\,dx$

$J = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}\,dx$

$I+J = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x} + \frac{\sin x}{\sin x+\cos x}\,dx$

$I+J = \int_0^{\frac{\pi}{2}} \frac{\cos x+\sin x}{\sin x+\cos x} = \int_0^{\frac{\pi}{2}} 1\,dx$

$I+J = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$I-J = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x} - \frac{\sin x}{\sin x+\cos x}$

$= \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{\sin x+\cos x}\,dx$

$= \int_0^{\frac{\pi}{2}} \frac{(\sin x+\cos x)'}{(\sin x+\cos x)}\,dx$

$= \ln\left|\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right| - \ln\lvert\sin 0 + \cos 0\rvert$

$= \ln(1) - \ln(1) = 0$

$I = J$

يعني

$I + J = \frac{\pi}{2}$

$I + I = \frac{\pi}{2}$ $\quad 2I = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

$J = \frac{\pi}{4}$ $\Rightarrow$ A

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Q_{27}

مجموعة النقط $H(z)$ بحيث

مجموعة النقط H بحيث $AH = BH$

هو واسط القطعة AB

$|z_B - z_A| = AB$ $\quad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$\frac{|z-4i|}{|z-(-2)|} = 1$ ومن $|z-4i| = |z-(-2)|$

$|z_H - z_A| = |z_H - z_B|$ / $A(4i), B(-2)$

$AH = BH$

اذن هاته المجموعة هي واسط قطعة $[AB]$

اذن هي مستقيم Ⓒ بحيث $A(0,4)$

$B(-2,0)$

مجموعة النقط $H(z)$ بحيث

$AH = r$ هي الدائرة التي مركزها

A وشعاعها r.

Q_{28}

الشكل الجبري للعدد:

$$z = \left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$$

$[r, \theta + 2k\pi] = [r, \theta]$

$[r, \theta]^n = [r^n, n\theta]$

$(1-i)^2 = -2i$

$$z = \frac{(1+i\sqrt{3})^{20}}{(1-i)^{20}} = \frac{\left[2, \frac{\pi}{3}\right]^{20}}{[(1-i)^2]^{20}}$$

$$= \frac{\left[2^{20}, \frac{2\pi}{3} + 6\pi\right]}{\left[2, \frac{\pi}{2}\right]^{10}} = \frac{\left[2^{20}, \frac{2\pi}{3}\right]}{\left[2^{10}, 10\frac{\pi}{2}\right]}$$

$$= \frac{\left[2^{20}, \frac{2\pi}{3}\right]}{\left[2^{10}, 6\pi\right]} = \frac{\left[2^{20}, \frac{2\pi}{3}\right]}{\left[2^{20}, \pi\right]}$$

$$= \left[\frac{2^{20}}{2^{10}}, \frac{2\pi}{3}\right]$$

$$z_{25} = 2^{10}\left(\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right)$$

$$= 2^{10}\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 2^9(1 - i\sqrt{3})$$

$$= 512(1 - i\sqrt{3}) \Rightarrow C$$

Q_{29}

$$h(x) = \sqrt{\frac{2-x}{2+x}}$$

معادلة المماس لـ C_f في النقطة

ذات الأفصول 1:

معادلة المماس لـ C_f في النقطة ذات

الأفصول x_0 هي

$$y = f'(x_0)(x - x_0) + f(x_0)$$

$$y = f'(1)(x-1) + f(1)$$

$$y = -\frac{2\sqrt{3}}{9}(x-1) + \frac{1}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{9}\left(-2x + 2 + \frac{9}{3}\right)$$

$$= \frac{\sqrt{3}}{9}(-2x + 5)$$

$$\left(\sqrt{U(x)}\right)' = \frac{(U(x))'}{2\sqrt{U(x)}}$$

$$* \left(\frac{U(x)}{V(x)}\right)' = \frac{U'(x)V(x) - U(x)V'(x)}{(V(x))^2}$$

$$(ax+b)' = a$$

$$\left(\frac{ax+b}{cx+d}\right)' = \frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2}$$

$$f'(x) = \left(\sqrt{\frac{-x+2}{x+2}}\right)' = \frac{\left(\frac{-x+2}{x+2}\right)'}{2\sqrt{\frac{-x+2}{x+2}}}$$

$$f'(x) = \frac{\begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix}}{2\sqrt{\frac{-x+2}{x+2}}(x+2)^2} = \frac{-4}{2(x+2)^2\sqrt{\frac{-x+2}{x+2}}}$$

Q30 المستوى (P) ذو المعادلة

$x-2y+2z-2=0$

الفلكة (S) ذات المعادلة

$x^2+y^2+z^2-2x+2z+1=0$

المسافة بين مركز الفلكة (S) والمستوى (P)

* المعادلة الفلكة ذات مركز $\Omega(a,b,c)$ وشعاع r

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

$$P: ax+by+cz+d=0$$

$$A(x_A, y_A, z_A)$$

$$d(A,(P))=\frac{|ax_A+by_A+cz_A+d|}{\sqrt{a^2+b^2+c^2}}$$

$x^2-2x=$

"2014"

Q21:

حل المعادلة

$$\ln(x+3)+\ln(x+2)=\ln(x+1)$$

$$\ln(a)+\ln(b)=\ln(ab)$$

$$\ln A=\ln B \iff A=B$$

$$\ln(x+3)(x+2)=\ln(x+1)$$

$$x^2+5x+6=x+1$$

$$x^2+4x-5=0$$

$$\Delta=36 \quad \begin{cases} x_1=\frac{-4-6}{2}=-5 \notin D_f \\ x_2=\frac{-4+6}{2}=1 \end{cases}$$

$$S=\{1\}$$

$$-\cos\theta+i\sin\theta=\cos(\pi-\theta)+i\sin(\pi-\theta)$$

$$-\cos\theta-i\sin\theta=\cos(\pi+\theta)+i\sin(\pi+\theta)$$

$$\cos\theta-i\sin\theta=\cos(-\theta)+i\sin(-\theta)$$

Q22

$$S_{2014}=1+i+i^2+\dots+i^{2014}$$

$$1+b+b^2+\dots+b^n=\frac{b^{n+1}-1}{b-1}$$

$$\boxed{\begin{aligned} i^{4k}&=1\\ i^{4k+1}&=i\\ i^{4k+2}&=-1\\ i^{4k+3}&=-i \end{aligned}}$$

$$1+i+i^2+\dots+i^{2014}=\frac{i^{2014+1}-1}{i-1}$$

$$=\frac{i^{2015}-1}{i-1}$$

$$2015=503\times4+3$$

$$i^{2015}=-i$$

$S=$

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$S_{2017} = 1 + i + i^2 + \dots + i^{2017}$

$$S = \frac{i^{2018} - 1}{i - 1}$$

$2018 = (504) \times 4 + 2$

$i^{2018} = -1$

$$S_{2017} = \frac{-1-1}{-1-i} = \frac{2}{1+i} = \frac{2(1-i)}{1+1}$$

$= 1 - i$

Q_{23}

مجموعة النقط H حيث

$[\quad (1-z)(1+\bar{z}) \in \mathbb{R}$

$z \in \mathbb{R} \iff Im(z) = 0$

$z \in i\mathbb{R} \iff Re(\bar{z}) = 0$

$(1-z)(1+\bar{z}) \in \mathbb{R}$

$\iff Im\left((1-z)(1+\bar{z})\right) = 0$

$\iff Im\left((1-x-iy)(1+x-iy)\right) = 0$

$\iff Im\left(1+x-iy-ix-x^2+ixy-y-ixy-y^2\right) = 0$

$Im \in 0$

$x + y - 1 = 0$

Q_{24}

$$v_{n+1} = \frac{5}{u_{n+1}} = \frac{5(3u_n+5)}{9u_n} = 3 + \frac{5}{u_n}$$

Q_{25}

	$-\infty$	-1		0		1		$+\infty$
	$-$		$-$	0	$+$		$+$	
	$+$	0	$-$		$-$	0	$+$	
	$-$	‖	$+$	‖	$-$	‖	$+$	

$D_f =]-1, 0[\cup]0, 1[$

$y = g'(1)(x-1) + g(1)$

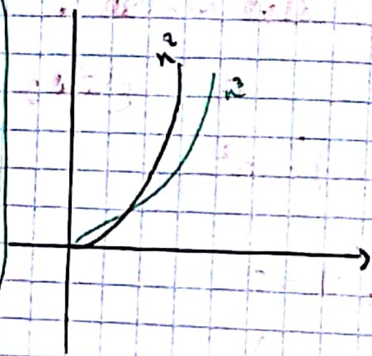
$g(x) = \left(f(\sin x\frac{\pi}{2}x)\right)'$

$= \frac{\pi}{2}\cos\left(\frac{\pi}{2}x\right)\left(f'(\sin\frac{\pi}{2}x)\right)$

$y = f(1)$

Q_{22}

$$A = \int_0^1 |f(x) - g(x)|\, dx$$



$$= \int_0^1 x^2 - x^3\, dx + \int_1^2 x^3 - x^2\, dx$$

$= \frac{1}{3} - \frac{1}{4} = \frac{3}{2}\ cm^2$

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"Cncrs 2015"

Kech

Q_{21}

$\lim_{x\to+\infty} \frac{(\ln(x))^m}{x^n} = 0^+$

$m>0$ و $n>0$

$\lim_{x\to 0^+} x^n (\ln(x))^m = 0$

$m>0$ و $n>0$

"$+\infty - \infty$"

نفك

$\lim_{x\to+\infty} x^m = 0$ $\quad m<0$

$\lim_{x\to 0^+} x^m = +\infty$ $\quad m<0$

$h(x) = x^m - (\ln(x))^2$

Faux $\lim_{x\to+\infty} h(x) = 0$ $\quad$ A

$\lim_{x\to+\infty} h(x) = \lim_{x\to+\infty} x^m - (\ln x)^2 = +\infty - \infty$

$= \lim_{x\to+\infty} x^m \left(1 - \frac{(\ln x)^2}{x^m}\right) = +\infty(1-0) = +\infty$

Faux $\lim_{x\to 0^+} h(x) = 0$ $\quad$ B.C

$\lim_{x\to 0^+} h(x) = \lim x^m - (\ln(x))^2$

$= \lim_{x\to 0^+} x^m \left(1 - x^{-m}(\ln x)^2\right)$ $\quad m<0$, $-m>0$

$= \lim_{x\to 0^+} \frac{1}{x^{-m}} \left(1 - x^{-m}(\ln x)^2\right)$

$= +\infty(1-0) = +\infty$

Faux $\lim_{x\to+\infty} h(x) = 0$ $\quad m<0$ $\quad$ D

$\lim_{x\to+\infty} h(x) = \lim_{x\to+\infty} x^m - (\ln x)^2$

$= 0 - (+\infty)^2 = -\infty$

Vrai $\lim_{x\to+\infty} h(x) = +\infty$ $\quad m>0$ $\quad$ E

Q_{22} $\quad u_n = \frac{(-1)^n}{n^2}$ $\quad n\in\mathbb{N}^*$

A رتيبة :

$u_1 = \frac{(-1)^1}{1^2} = -1$

$u_2 = \frac{(-1)^2}{2^2} = \frac{1}{4}$

$u_3 = \frac{(-1)^3}{3^2} = \frac{-1}{9}$

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$$U_{n+1} - U_n = \frac{(-1)^{n+1}}{(n+1)^2} - \frac{(-1)^n}{n^2}$$

$$= \frac{(-1)^{n+1}}{(n+1)^2} + \frac{(-1)(-1)^n}{n^2}$$

$$= (-1)^{n+1}\left(\frac{1}{(n+1)^2} + \frac{1}{n^2}\right)$$

لكي تحقق

(U_n) رتيبة يجب أن

يكون للفرق $U_{n+1} - U_n$ اشارة

قارة لكن $(-1)^{n+1}$ يكون موجبا

اذا كان n فردي

و $(-1)^{n+1}$ سالبا اذا كان n زوجي

اذن $\not\subset$ (F)

F (D)

F (E)

$U_{n+1} = 2U_n + 1 = f(U_n)$ تراجعية

$U_n = \frac{2n+1}{n+2} = f(n)$ صريحة

$l \in \mathbb{R}$

B

$\lim_{n\to\infty} U_n = l \iff U_n$ متقاربة

$$\lim_{n\to+\infty} \frac{(-1)^n}{n^2} = \begin{cases} \lim_{n\to+\infty} \frac{1}{n^2} = 0 & n \text{ زوجي} \\ \lim_{n\to+\infty} \frac{-1}{n^2} = 0 & n \text{ فردي} \end{cases}$$

$\lim_{n\to+\infty} \frac{(-1)^n}{n^2} = 0$ U_n متقاربة

Q23

(A) الجزء الحقيقي $(1-i)^5$ هو $\sqrt{2}$

$$(1-i)^5 = ((1-i)^2)^2 (1-i)$$

$$= (-2i)^2 (1-i)$$

$$= -4(1-i) = -4 + 4i$$

$Re(1-i)^5 = -4$

(A) est fausse

$$(1+i)^{20} = ((1+i)^2)^{10} = (2i)^{10} = 1024 \text{ (B)}$$

$Im = 0$

B خاطئة

$(1+i)^{20} \in \mathbb{R}$ $\mathbb{C}$

$Im(1+i)^{20} = 0$

اذن $(1+i)^{20} \in \mathbb{R}$

نفس $\mathbb{C}$

① المعادلة $z^4 - 1 = 0$ تقبل حليت

1 و -1 في $\mathbb{R}$

وتقبل 4 حلول في $\mathbb{C}$ وهي $1, -1$

i و $-i$

⊛ كل معادلة في $\mathbb{C}$ من الدرجة n تقبل n حل.

تعريف ⊛ z جذر من الرتبة n للعدد 1 $\Leftrightarrow$

$z^n - 1 = 0$

خاصية: للعدد z, n جذر من الرتبة n في $\mathbb{C}$

يعني للعدد 1 جذر من الرتبة n وهي

$z_k = [1, \frac{2k\pi}{n}]$, $k \in \{0, 1, \ldots, (n-1)\}$

⊛ مثال

حدد الجذور من الرتبة 4 للعدد 1

حل المعادلة $z^4 = 1$

الجذور هي $z_k = [1, \frac{2k\pi}{4}]$; $k \in \{0, 1, 2, 3\}$

$z_0 = [1, 0] = 1$; $z_2 = [1, \pi] = -1$

$z_1 = [1, \frac{\pi}{2}] = i$; $z_3 = [1, \frac{3\pi}{2}] = -i$

P_{24}

$$f \begin{cases} f(x) = e^x & si \ x < 0 \\ f(x) = \cos x & si \ x \geq 0 \end{cases}$$

المعادلة $f(x)$ تقبل 3 حلول في $]-\infty, 2\pi[$

$\cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$

$x \in]-\infty, 0[$ إذا كان

$f(x)=0$

$e^x=0$ $\quad \forall x \in \mathbb{R} : e^x > 0$

$S_1 = \emptyset$

$x \in [0, 2\pi]$ إذا كان

$f(x)=0$

$\cos x = 0 \iff x = \frac{\pi}{2} + K\pi$

نؤطر $\frac{\pi}{2} + K\pi$

$0 \leq \frac{\pi}{2} + K\pi \leq 2\pi$

$0 \leq \frac{1}{2} + K \leq 2$

$-\frac{1}{2} \leq K \leq \frac{3}{2}$

$K=0$ $\quad K=1$

إذن للمعادلة $f(x)=0$ حلين في $]-\infty, 2\pi]$

إذن A خاطئة

ومن هنا يتبين لنا أن المعادلة $f(x)=0$ لها حل وحيدا هو $\frac{\pi}{2}$ في المجال $]-\infty, \pi]$

إذن الإقتراح E هو الصحيح

$$\cos a = \cos b \iff \begin{cases} a = b + 2K\pi \\ a = -b + 2K\pi \end{cases}$$

$$\sin a = \sin b \iff \begin{cases} a = b + 2K\pi \\ a = (\pi - b) + 2K\pi \end{cases}$$

f ليست متصلة في 0.

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \cos x = \cos(0) = 1$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} e^x = e^0 = 1$

اذا كانت f قابلة للاشتقاق في x_0 فإن f متصلة في x_0

اذا كانت f غير متصلة في x_0 فإنها غير قابلة للاشتقاق

Q25

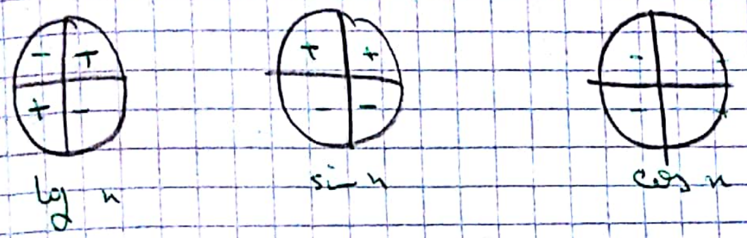
$$\int_2^e \frac{1}{x\ln(x)}\,dx = -2 \quad \text{Faux}$$

(خاصية 1)

$$\forall x \in [a,b] \quad f(x) \geq 0 \implies \int_a^b f(x)\,dx \geq 0$$

خاصية 2:

$$\forall x \in [a,b] \quad f(x) \leq 0 \implies \int_a^b f(x)\,dx \leq 0$$



$$\int_0^{\pi/4} tg\,x\,dx = -\frac{1}{2}\ln 2$$

$$\forall x \in \left[0, \frac{\pi}{4}\right] \implies tg\,x \geq 0$$

B

Q26

$$f(x) = \frac{1}{1+x^2} \quad \text{et} \quad g(x) = \int_x^{x+1} f(t)\,dt$$

$$f(\mathbb{R}) =]0,1] \qquad A$$

$$f(I) = \{f(x) \,/\, x \in I\}$$

$$x \in \mathbb{R} \iff f(x) \in f(I)$$

$$x \in \mathbb{R} \quad x^2 \geq 0$$

$$\iff 1 + x^2 \geq 1$$

$$\iff 0 < \frac{1}{1+x^2} \leq 1$$

$$\iff 0 < f(x) \leq 1$$

$$\iff f(\mathbb{R}) =]0,1]$$

2: طريقة

$$f'(x) = -\frac{(1+x^2)'}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2}$$

x	$-\infty$		0		$+\infty$
$f'(x)$		$+$	0	$-$	
f	0	$\nearrow$	1	$\searrow$	0

$g'(x) = f(x) - f(x+1)$ خطأ C

اذا كانت

f متصلة على المجال $[x, x+a]$

$a>0$ فإنها تقبل دالة أصلية F على

$[x, x+a]$

$$g(x) = \int_x^{x+a} f(t)\,dt$$

$$= [F(t)]_x^{x+a}$$

$$g(x) = F(x+a) - F(x)$$

$$g'(x) = (F(x+a))' - (F(x))'$$

$$\Rightarrow g'(x) = (F(x+a))' - F(x)'$$

$$= (x+a)' F(x+a) - f(x)$$

$$= 1\, f(x+a) - f(x)$$

$$= f(x+a) - f(x)$$

$\forall x \in \mathbb{R}$ $\quad g(x) < 0$ $\quad$:D

$\forall x \in [a, b]$

$f(x) \geq 0 \quad \Rightarrow \int_a^b f(x)\,dx \geq 0$

$a < b$

لدينا $\forall t \in [x, x+1]$

$f(t) = \frac{1}{1+t^2} > 0 \quad \Rightarrow g(x) = \int_x^{x+1} f(t)\,dt > 0$

$x < x+1$

$g(x) > 0$

$0 \leq g(x) < \frac{1}{2}$ لكل . E

من خلال ما سبق

$$0 < \frac{1}{1+t^2} \leq 1$$

$$0 < \int_x^{x+1} \frac{1}{1+t^2} < \int_x^{x+1} 1\,dt$$

$$0 < g(n) < [t]_n^{n+1}$$
$$0 < g(n) < (n+1)-n$$
$$0 < g(n) < 1$$

Faux E

Q_{14}

$p \in \mathbb{N}^*$ و $n \in \mathbb{N}^*$

+	P	I
P	P	I
I	I	P

×	P	I
P	P	P
I	P	I

E صحيحة

Q_{28}

A $\int_0^{\frac{\pi}{2}} \frac{2}{\cos^2(x)}\,dx = 1-\sqrt{2}$

$a < b$ $\quad \int_a^b \frac{1}{\cos^2 x}\,dx = [\operatorname{tg} x]_a^b$

بما أن $\frac{2}{\cos^2 x} > 0$

$\left(0 < \frac{\pi}{2}\right)$

$$\int_0^{\frac{\pi}{2}} \frac{2}{\cos^2 x}\,dx > 0$$

وبما أن $1-\sqrt{2} < 0$

فإن A خاطئة.

B. $\quad C = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{2}{\cos^2 x}\,dx = 4(\sqrt{2}-1)$

$$= 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{\cos^2 x}\,dx = 2\left[\operatorname{tg} x\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= 2\left[\operatorname{tg}\frac{\pi}{4} - \operatorname{tg}\frac{-\pi}{4}\right]$$

$$= 2[1-(-1)] = 4$$

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C

$$\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{2}{\cos^2(x)}\,dx$$

$$= 2\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{1}{\cos^2(x)}\,dx$$

$$= 2\left[tg(x)\right]_{-\frac{\pi}{8}}^{\frac{\pi}{8}}$$

$$= 2\left[tg\frac{\pi}{8} - tg\left(-\frac{\pi}{8}\right)\right]$$

$$= 2\left[2\,tg\frac{\pi}{8}\right]$$

$$tg(a+b) = \frac{tg(a)+tg(b)}{1-tg(a)\,tg(b)}$$

$$tg(2a) = \frac{2\,tg\,a}{1-tg^2(a)}$$

$$a = \frac{\pi}{8} \Rightarrow tg\left(\frac{2\pi}{8}\right) = \frac{2\,tg\frac{\pi}{8}}{1-tg^2\left(\frac{\pi}{8}\right)}$$

$$\Rightarrow 1 = \frac{2\,tg\left(\frac{\pi}{8}\right)}{1-tg^2\left(\frac{\pi}{8}\right)} \Leftrightarrow 1 - tg^2\frac{\pi}{8}) = 2\,tg\frac{\pi}{8}$$

$$tg^2\frac{\pi}{8} + 2\,tg\frac{\pi}{8} - 1 = 0$$

$$tg\frac{\pi}{8} = -1-\sqrt{2}$$

$$\Delta = 4+4 = 8$$

$$tg\frac{\pi}{8} = -\frac{2+\sqrt{8}}{2} = -1+\sqrt{2}$$

$$0 < \frac{\pi}{8} < \frac{\pi}{2} \Rightarrow tg\left(\frac{\pi}{8}\right) = \sqrt{2}-1$$

P29

$$U_n = \frac{e^n}{n^n} \quad \text{et} \quad V_n = \ln(U_n)$$

A. $\lim U_n = \lim V_n$

$$\lim U_n = \lim \frac{e^n}{n^n}$$

$$= \lim_{n\to+\infty} \left(\frac{e}{n}\right)^n$$

$$= \lim_{n\to+\infty} e^{n\ln\left(\frac{e}{n}\right)}$$

$$= e^{-\infty} = 0$$