Les astuces pour calculer les limites

$\frac{1}{1-x} = = (1-x)^{-1} = 1+x$

$\boxed{(1+x)^{\alpha} = 1+\alpha x}$ | $\frac{1}{1+x} = 1-x$

$(1-x)^{\alpha} = 1-\alpha x$

$\boxed{\frac{1}{1-x} = 1+x}$ | $\boxed{\frac{1}{1+x} = 1-x}$

$\boxed{\sqrt{1+x} = (1+x)^{1/2} = 1+\frac{x}{2}}$

$\boxed{\sqrt{1-x} = 1-\frac{x}{2}}$

$\boxed{(1+U(x))^{\alpha} = 1+\alpha . U(x)}$

$$(2+U(x))^{\alpha} = \left(2\left(1+\frac{U(x)}{2}\right)\right)^{\alpha}$$
$$= 2^{\alpha} \times \left(1+\frac{U(x)}{2}\right)^{\alpha}$$
$$= 2^{\alpha}\left(1+\alpha . \frac{U(x)}{2}\right)$$

$(1+x)^{\alpha} = 1+\alpha x$ $\quad$ $(1-x)^{\alpha} = 1-\alpha x$

$\frac{1}{1+x} = 1-x$ $\quad$ $\frac{1}{1-x} = 1+x$

$\boxed{\ln(1+x) = x} \Leftarrow \lim\limits_{x\to 0} \frac{\ln(1+x)}{x} = 1$

$\boxed{\ln(1-x) = -x}$

des exemples:

② $\lim\limits_{x\to 0} \frac{\sqrt{x+x^2} - \sqrt{x}}{\sqrt{3x}\,\ln(1+x)}$

$\frac{\sqrt{x}.\sqrt{1+x} - \sqrt{x}}{\sqrt{3x}.x}$

$\lim\limits_{x\to 0} \frac{\sin x}{x} = 1 \Longleftrightarrow \boxed{\sin x = x}$

$\lim\limits_{x\to 0} \frac{\tan x}{x} = 1 \Longleftrightarrow \boxed{\tan x = x}$

$\lim\limits_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \Longleftrightarrow 1-\cos x = \frac{x^2}{2}$ $\quad \boxed{\cos x = 1-\frac{x^2}{2}}$

Régle de l'hopital:

$\lim\limits_{x\to x_0} \frac{f(x)}{g(x)} = \text{"}\frac{0}{0}\text{"}$

$\lim\limits_{x\to x_0} \frac{f'(x)}{g'(x)} = \ldots$

$\lim\limits_{x\to 0} \frac{e^x-1}{x} = 1$ $\quad \boxed{e^{u(x)} = 1+U(x)}$

عندما يكون x قريبا من 0 فإن:

$e^x - 1 \simeq x$

$\boxed{e^x = 1+x}$

$\boxed{e^{u(x)} \simeq 1+U(x)}$

أس قريب من 0

$U(x) \xrightarrow[x\to x_0]{} 0$

$\boxed{e^{1-x} = 2-x}$ $\quad x\to 1$

① $\lim\limits_{x\to 0} \frac{e^{10x} - e^{7x}}{x} = \lim\limits_{x\to 0} \frac{1+10x-1-7x}{x}$

$= \lim\limits_{x\to 0} \frac{3x}{x} = 3$

$\Leftrightarrow \boxed{\frac{10e^{10x} - 7e^{7x}}{1} = 3}$ Regle de l'hopital

⑪ $\lim\limits_{x\to 0} \frac{e^{2x}-1}{\tan x} = \frac{e^{2x}-1}{2x} \times \frac{1}{\frac{\tan x}{2x}}$

$\boxed{\frac{\cancel{1}+2x-\cancel{1}}{x} = 2}$ $\quad = 2$

$$\lim_{x\to+\infty}\left(1-\frac{1}{x}\right)^x$$

$$e^{x\times -\frac{1}{x}} = \boxed{e^{-1}}$$

$$\lim_{x\to+\infty}\left(1+\frac{2}{x^2}\right)^x = e^{x\ln\left(1+\frac{2}{x^2}\right)}$$
$$= e^{x\times\frac{2}{x^2}}$$
$$= e^{\frac{2}{x}} = 1$$

7) $$\lim_{x\to 0}\frac{1-\sqrt{1+\sin x}}{x}$$

$$= \frac{1-1-\sin x}{x\left(1+\sqrt{1+\sin x}\right)} = \boxed{-\frac{1}{2}}$$

$$= \frac{\cancel{1}-\cancel{1}-\sin x}{2x}$$ $\sqrt{1+\sin x} = (1+u(x))^{\frac{1}{2}}$ $= 1+\frac{u(x)}{2}$

$$= -\frac{1}{2}$$

8) $$\lim_{x\to\infty} e^{-x}\ln(1-e^x)$$

$$= e^{-x}\times(-e^x) = \boxed{-1}$$ $\ln(1+u(x)) = u(x)$

9) $$\lim_{x\to 0}\frac{1}{x}\ln\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \frac{1}{x}\left(\ln(1-x^2)-\ln(1+x^2)\right)$$

$$= \frac{1}{x}\left(-x^2-x^2\right) = \frac{-2x^2}{x} = -2x = \boxed{0}$$

10) $$\lim_{x\to+\infty}\left(\sqrt{x+1}-\sqrt{x}\right)\ln x$$

$$= \sqrt{x}\left(\sqrt{1+\frac{1}{x}}-1\right)\ln x$$

$$= \sqrt{x}\left(\cancel{1}+\frac{1}{2x}-\cancel{1}\right)\ln x$$

$$= \frac{2\ln\sqrt{x}}{2\sqrt{x}} = 0$$ $\sqrt{1+x} = 1+\frac{x}{2}$

$$\frac{\sqrt{x}\sqrt{1+x}-\sqrt{x}}{\sqrt{3}x\cdot x} = \frac{1+\frac{x}{2}-1}{\sqrt{3}\cdot x} = \boxed{\frac{1}{2\sqrt{3}}}$$

3) $$\lim_{x\to 0}\frac{\ln(\cos(2x))}{\ln(\cos(3x))}$$ $\boxed{e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}}$

Devellopement limite

$\cos x = 1-\frac{x^2}{2}$

$\cos u(x) = 1-\frac{u(x)}{2}$

$$= \frac{\ln\left(1-\frac{4x^2}{2}\right)}{\ln\left(1-\frac{9x^2}{2}\right)}$$

$$= \frac{-\frac{4x^2}{2}}{-\frac{9x^2}{2}} = \frac{4}{9}$$

4) $$\lim_{x\to 0^+}\frac{\ln(x)+x^2}{\ln(x+x^2)} = \frac{\ln(x)+x^2}{\ln(x(1+x))}$$
$$= \frac{\ln(x)+x^2}{\ln x+\ln(1+x)}$$
$$= \frac{\ln(x)+\cancel{x^2}}{\ln(x)+\cancel{x}} = 1$$

5) $$\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[4]{1+2x}}{x}$$

$\boxed{\sqrt[n]{1+x}^{\,p} = (1+x)^{\frac{p}{n}} = 1+\frac{p}{n}\times x}$

$$\frac{1+\frac{x}{3}-1-\frac{x}{2}}{x}$$

$$= \frac{1}{3}-\frac{1}{2} = -\frac{1}{6}$$

6) $$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x$$

$$e^{x\ln\left(1+\frac{1}{x}\right)}$$

$$e^{x\times\frac{1}{x}} = \boxed{e}$$

$\ln\left(1+\frac{1}{x}\right) = \frac{1}{x}$

$\ln\left(1+\frac{a}{x}\right) = \frac{a}{x}$

$i=1$ $\qquad \underbrace{+1 \dots +1}_{n \text{ fois}}$ $\qquad z^9 = [(\sqrt{2})^9 ; 9\times\frac{\pi}{4}]$

Limites

$$\sum_{k=0}^{n} \ln(U_k) = \ln\left(\prod_{k=0}^{n} U_k\right)$$

$$\exp\left(\sum_{k=0}^{n} U_k\right) = \prod_{k=0}^{n} U_k \cdot V_k$$

$$\int_0^3 \frac{dx}{3+2^x}$$

Changement de variable:

$$\int_0^1 2x(x^2+1)^4\,dx$$

$$(x^2+1)' = 2x$$

on pose $t = x^2+1 \Leftrightarrow x = \sqrt{t-1}$

$$dt = 2x\,dx$$

si $x=0 \Rightarrow 0+1 = t \Rightarrow t=1$

si $x=1 \Rightarrow 1^2+1 = t \Rightarrow t=2$

$$\Rightarrow \int_1^2 t^4\,dt = \left[\frac{1}{5}t^5\right]_1^2 = \frac{32}{5} - \frac{1}{5} = \frac{31}{5}$$

$$I = \int_0^3 \frac{dx}{3+2^x}$$

$\boxed{2^x = e^{x\ln(2)}}$

pose $t = 2^x$

pour $x=0 \Rightarrow t=1$

$x=3 \Rightarrow t=8$

$$dt = (2^x)'\,dx = \ln 2 \cdot e^{x\ln 2}\,dx = \ln 2 \cdot 2^x \cdot dx = t\ln 2\,dx$$

$$I = \int_1^8 \frac{1}{3+t} \cdot \frac{1}{t\ln 2}\,dt$$

$$= \frac{1}{\ln 2}\int_1^8 \frac{1}{t(t+3)}\,dt$$

$$f(x) = \frac{1}{t(t+3)} = \frac{\alpha}{t} + \frac{\beta}{t+3}$$

Q9: $z = a+ib$

$\boxed{z^2 = 5-12i}$

$z = a+ib \Leftrightarrow z^2 = a^2-b^2+2iab = 5-12i$

$|z^2| = |z|^2 = \sqrt{a^2+b^2}^2 = a^2+b^2$

$$\Rightarrow \begin{cases} a^2+b^2 = \sqrt{5^2+(12)^2} = 13 \\ a^2-b^2 = 5 \Rightarrow 2a^2 = 18 \\ 2ab = -12 \end{cases}$$

$a^2 = 9$

$\boxed{a=3 ; a=-3}$

$\Rightarrow 2b^2 = 8 \Rightarrow b^2 = 4$

$\Rightarrow b = 2$ ou $b = -2$

$2ab = -12 \Rightarrow ab = -6$

ab = quantité $Re(z_1) \cdot Im(z_2)$

Q10: $z = \left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$

partie imaginaire?

$z = \left(\frac{z_1}{z_2}\right)^{20}$ $\quad z_1 = 1+i\sqrt{3}$

$\Rightarrow |z_1| = 2$

$\Rightarrow \cos\theta = \frac{1}{2}$ $\quad \sin\theta = \frac{\sqrt{3}}{2}$

$\boxed{\theta = \frac{\pi}{3}}$

$z_2 = 1-i \Rightarrow |z_2| = \sqrt{2}$

$\cos\theta = \frac{\sqrt{2}}{2}$ $\qquad \sin\theta = -\frac{\sqrt{2}}{2}$

$\Rightarrow \theta = -\frac{\pi}{4}$

$$\Rightarrow z = \left(\frac{2e^{i\frac{\pi}{3}}}{\sqrt{2}\cdot e^{-\frac{\pi}{4}}}\right)^{20} = \frac{2^{20}}{(\sqrt{2})^{20}} \cdot \frac{e^{\frac{i20\pi}{3}}}{e^{-\frac{20\pi}{4}\pi}}$$

$$\Rightarrow 2^{10} \cdot e^{-\frac{i\pi}{3}} = -512\sqrt{3}$$

$$\lim_{n\to+\infty} \frac{1}{n^2} \sum_{k=1}^{n} E(7k)$$

$x-1 < E(x) \leqslant x \qquad 3{,}2 \to 3$

on $x = 7k$

$\Rightarrow 7k - 1 < E(7k) \leqslant 7k$

$\sum_{k=1}^{n} (7k-1) \leqslant \sum_{k=1}^{n} E(7k) \leqslant \sum_{k=1}^{n} 7k$

$\Rightarrow 7\,\frac{n(n+1)}{2} - n < \sum_{k=1}^{n} E(7k) \leqslant 7\,\frac{n(n+1)}{2}$

$\Rightarrow 7\cdot\frac{n^2+n}{2n^2} - \frac{n}{n^2} < \frac{1}{n^2}\sum_{k=1}^{n} E(7k) \leqslant 7\cdot\frac{n^2+n}{2n^2}$

$\lim_{x\to+\infty} \Rightarrow \frac{7}{2} \leqslant \lim_{n\to+\infty} \frac{1}{n^2} \sum_{k=1}^{n} E(7k) \leqslant \frac{7}{2}$

$\Rightarrow \lim_{n\to+\infty} \frac{1}{n^2} \sum_{k=1}^{n} E(7k) = \frac{7}{2}$

Q8 : $\lim_{n\to+\infty} \left(\sqrt[n]{2+(-1)^n}\right)$

$= \lim_{n\to+\infty} (2+(-1)^n)^{\frac{1}{n}} = \lim_{n\to+\infty} e^{\frac{1}{n}\ln(2+(-1)^n)}$

$= e^0 = 1$

$1 \leqslant (-1)^n \leqslant 1$

$3 \leqslant (-1)^n + 2 \leqslant 3$

$\sqrt{3} \leqslant \sqrt[n]{(-1)^n+2} \leqslant \sqrt[n]{3}$

$\Rightarrow \lim_{n\to+\infty} \Rightarrow 1 \leqslant \lim_{n\to+\infty} \sqrt[n]{(-1)^n+2} \leqslant 1$

somme de Reimon

$$\lim_{n\to+\infty} \frac{b-a}{n} \sum_{k=1}^{n} f\left(a+k\frac{b-a}{n}\right)$$

$$\sum_{i=1}^{10} \frac{1}{k(k+1)}$$

$= \sum_{k=1}^{10} \left(\frac{1}{k} - \frac{1}{k+1}\right)$

$= \sum_{k=1}^{10} \frac{1}{k} - \sum_{k=1}^{10} \frac{1}{k+1}$

$= \left(1 + \frac{1}{2} + \frac{1}{3} + \cancel{\cdots} + \frac{1}{10}\right) - \left(\frac{1}{2} + \cdots + \frac{1}{11}\right)$

$= 1 - \frac{1}{11} = \boxed{\frac{10}{11}}$

$f(x) = x^2 \sin\left(\frac{1}{x}\right) \qquad x = \frac{1}{t}$

$\lim_{x\to 0} f(x) = \lim_{x\to+\infty} \frac{1}{t^2}\sin(t)$

$= \lim_{t\to\infty} \frac{\sin(t)}{t^2} = 0$

$-\frac{1}{t^2} \leqslant \frac{\sin(t)}{t} \leqslant \frac{1}{t^2}$

$\boxed{\lim_{x\to 0} f(x) = 0 = f(0)} \Rightarrow f$ est continue en 0

$\hookrightarrow$ continue en $\mathbb{R}$.

$f'(x) = \left(x^2 \sin\left(\frac{1}{x}\right)\right)'$

$= 2x \sin\left(\frac{1}{x}\right) + x^2 \cdot \left(\frac{1}{x^2}\right)\cdot\cos\left(\frac{1}{x}\right)$

$= 2x \sin\left(\frac{1}{x}\right) \mp \cos\left(\frac{1}{x}\right)$

$\lim_{x\to 0} f'(x) = \underbrace{\lim_{x\to 0} 2x \sin\left(\frac{1}{x}\right)}_{0} \quad \lim_{x\to 0} \frac{-\cos 1}{x}$

developpement limite

Formule de Taylor :

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$2! = 2 \times 1 = 2$

$1! = 1$

$0! = 1$

$$f(x) = e^x \Leftrightarrow \begin{cases} f'(x) = e^x \\ f''(x) = e^x \\ f'''(x) = e^x \end{cases} \Leftrightarrow \begin{cases} f'(0) = 1 \\ f''(0) = 1 \\ f'''(0) = 1 \end{cases}$$

$$\begin{cases} 1! = 1 \\ 2! = 2 \times 1 = 2 \\ 3! = 3 \times 2 \times 1 = 6 \end{cases}$$

donc : $f(x) = 1 + x \times 1 + \frac{x^2}{2} \times 1 + \frac{x^3}{3} \times 1$

$= 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + x^3 \varepsilon(x)$

$$\lim_{x \to 0} \frac{\ln(1-x) - x}{x^2}$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2} f''(0) + x^2 \varepsilon(x)$$

$$f(x) = \ln(1-x) \Rightarrow \begin{cases} f'(x) = \frac{-1}{1-x} \\ f''(x) = -\frac{1}{(1-x)^2} \end{cases}$$

$$\Leftrightarrow \begin{cases} f'(0) = -1 \\ f''(0) = -1 \\ f(0) = 0 \end{cases}$$

et $\begin{cases} 1! = 1 \\ 2! = 2 \times 2 \end{cases}$ $\Rightarrow f(x) = 0 + x(-1) + \frac{x^2}{2} \times (-1) + \frac{x^3}{3} \times (0)$

$= -x - \frac{1}{2} x^2 + x^2 \varepsilon(x)$

$$B = \int_0^{-\ln 2} \frac{dx}{2e^x + 1}$$

on pose $t = e^{-x} \Rightarrow \ln t = -x$

$x = -\ln t$

$dx = -\frac{dt}{t}$

$$\begin{cases} x = -\ln 2 \\ x = 0 \end{cases} \quad \begin{cases} t = +2 \\ t = 1 \end{cases}$$

$$B = \int_1^2 \frac{-\frac{dt}{t}}{2e^{-\ln t} + 1}$$

$$= -\int_1^2 \frac{dt/t}{\frac{2}{t} + 1} = -\int_1^2 \frac{dt}{2+t}$$

$$= -[\ln|2+t|]_1^2$$

$$C = \int_1^4 \frac{x-1}{\sqrt{x}+1}$$

$$\sum_{k=0}^{n} C_n^k = 2^n$$

$t = \sqrt{x} \Rightarrow x = t^2$

$dx = 2t \cdot dt$

$x \begin{cases} 4 \\ 1 \end{cases} \to t \begin{cases} 2 \\ 1 \end{cases}$

$$C = \int_1^2 \frac{t^2 - 1}{t+1} \cdot 2t \cdot dt$$

$$= 2 \int_1^2 \frac{t(t^2-1)}{t+1} dt = 2 \int_1^2 \frac{t(t-1)(t+1)}{t+1} dt$$

$$= 2 \left[\frac{t^3}{3} + \frac{t^2}{2}\right]_1^2$$

$$\frac{1}{n}\sum_{k=0}^{n-1}\frac{k}{\sqrt{4n^2-k^2}}$$

$$\frac{k}{\sqrt{4n^2-k^2}} = \frac{n\times\frac{k}{n}}{\sqrt{4n^2-n^2\left(\frac{k^2}{n^2}\right)}}$$

$$= \frac{n\times\left(\frac{k}{n}\right)}{n\sqrt{4-\left(\frac{k}{n}\right)^2}} = \frac{\left(\frac{k}{n}\right)}{\sqrt{4-\left(\frac{k}{n}\right)^2}}$$

$$f(x) = \frac{x}{\sqrt{4-x^2}}$$

$$S_n = \frac{1}{n}\sum_{k=0}^{n-1} f\left(\frac{n}{k}\right) \xrightarrow[+\infty]{} \int_0^1 \frac{x}{\sqrt{4-x^2}}\,dx$$

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$= -\frac{1}{2}\int_0^1 \frac{-2x}{\sqrt{4-x^2}}\,dx$$

$$= -\frac{1}{2}\left[4-x^2\right]_0^1$$

changement de variable

$$A = \int_0^1 x\sqrt{x^2+1}\,dx$$

on pose $t = \sqrt{x^2+1}$; $dt = \frac{2x\,dx}{2\sqrt{x^2+1}}$

$$dt = \frac{x}{\sqrt{x^2+1}}\,dx \Rightarrow t.dt = x\,dx$$

② $\begin{cases} \text{si } x = 1 \\ \text{si } x = 0 \end{cases} \Rightarrow \begin{cases} t = \sqrt{2} \\ t = 1 \end{cases}$

$$A = \int_1^{\sqrt{2}} t\; t.dt$$

$$= \int_1^{\sqrt{2}} t^2.dt = \left[\frac{t^3}{3}\right]_1^{\sqrt{2}}$$

$$= \frac{(\sqrt{2})^3}{3} - \frac{1}{3} = \frac{2\sqrt{2}}{3} - \frac{1}{3}$$

somme de Reiman

$$\lim_{n\to+\infty}\frac{b-a}{n}\sum_{k=1}^{n} f\left(a+k.\frac{b-a}{n}\right) \to \int_a^b f(t)\,dt$$

$$\sum_{k=1}^{n}\frac{n}{n^2+k^2}$$

$$\frac{n}{n^2+k^2} = \frac{n}{n^2\left(1+\frac{k^2}{n^2}\right)} = \frac{1}{n\left(1+\frac{k^2}{n^2}\right)}$$

$$\sum_{k=1}^{n}\frac{n}{n^2+k^2} = \sum_{k=1}^{n}\frac{1}{n\left(1+\left(\frac{k}{n}\right)^2\right)}$$

$$= \frac{1}{n}\sum_{k=1}^{n}\frac{1}{1+\left(\frac{k}{n}\right)^2}$$

$$= \frac{1}{n}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)$$

donc $\lim_{n\to+\infty} S_n = \int_0^1 \frac{1}{1+x^2}\,dx$

$$= \left[\text{Arctan}\,x\right]_0^1 = \text{Arctan}\,1 - \text{Arctan}\,0$$

$$= \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$$

$$\sum_{k=1}^{n}\frac{k}{n^2+k^2}$$

$$\frac{k}{n^2+k^2} = \frac{n\times\frac{k}{n}}{n^2+\left(\frac{k}{n}\right)^2\times n^2} = \frac{1}{n}\times\frac{\left(\frac{k}{n}\right)}{1+\left(\frac{k}{n}\right)^2}$$

$$f(x) = \frac{x}{1+x^2}$$

$$\int_0^1\frac{x}{1+x^2} = \frac{1}{2}\int_0^1\frac{2x}{1+x^2}\,dx = \frac{1}{2}\left[\ln(1+x^2)\right]$$

$$= \frac{1}{2}(\ln 2 - \ln 1)$$

ensam

* La période T

f de periode T si $f(x)=f(x+T)$

$\boxed{\cos(a+b)=\cos a.\cos b-\sin a.\sin b}$

$\boxed{\sin(a+b)=\sin a\cos b+\sin b\cos a}$

$(\sin x)'=-\cos x$

Ex :

$f(x)=\cos x$ est 2π periodique car

$f(x+2\pi)=\cos(x+2\pi)$

$=\cos x\cos 2\pi-\sin x\sin 2\pi$

$=\cos x=f(x)$

comment Trouver T ?

$\Rightarrow f(x)=\sin(5x)$

$\Rightarrow f(x+T)=\sin x(5x+5T)$

$=\sin(5x).\cos(5T)+\cos(5x)\sin 5T$

il faut $\Rightarrow \begin{cases}\cos(5T)=1\\ \sin(5T)=0\end{cases} \Rightarrow 5T=0+2k\pi$

$T=2k\frac{\pi}{5}$

$\begin{cases}\cos x=1\\ \sin x=0\end{cases} \Leftrightarrow \boxed{x=0}+2k\pi$

alors $f(x)$ est $\frac{2\pi}{5}$ periodique

$f(x)=\sin\frac{x}{2}+\cos x$

$f(x+T)=\sin\left(\frac{x}{2}+\frac{T}{2}\right)+\cos(x+T)$

$=\sin\frac{x}{2}.\cos\frac{T}{2}+\cos x.\sin\frac{T}{2}+\cos x\cos T-\sin x\sin T$

$\Leftrightarrow \begin{cases}\cos\frac{T}{2}=1\\ \cos T=1\end{cases}$ et $\begin{cases}\sin T=0\\ \sin\frac{T}{2}=0\end{cases}$

$\Leftrightarrow \begin{cases}\cos\frac{T}{2}=1\\ \sin\frac{T}{2}=0\end{cases}$ et $\begin{cases}\cos T=1\\ \sin T=0\end{cases}$

$\Rightarrow \frac{T}{2}=0+2k\pi$ $\quad T=0+2k\pi$

$\Rightarrow T_1=4k\pi$ $\quad T_2=2k\pi$

$T_1\cap T_2=4k\pi$ $\quad k\in\mathbb{Z}$

alors 4π Periodique

* Resoudre $\cos^4(4x)-\sin^4(4x)=1$

on sai $\cos^2 x+\sin^2(x)=1$

$\Leftrightarrow \cos^2(x)=1-\sin^2(x)$

$\cos^4(x)=(\cos^2 x)^2=(1-\sin^2(x))^2$

$(1-\sin^2 4x)^2-\sin^4 4x=1$

$\Rightarrow 1-2\sin^2 4x+\sin^4 4x-\sin^4 4x=1$

$\Rightarrow 1-2\sin^2 4x=1$

$\Rightarrow -2\sin^2 4x=0$

$\Leftrightarrow \sin^2 4x=0$

$\Leftrightarrow \sin 4x=0$

$\Rightarrow 4x=k\pi$

$\Rightarrow \boxed{x=k.\frac{\pi}{2}}$ avec $k\in\mathbb{Z}$

$(1+i)^9=a+ib$ (trouver a et b)

$(z)^n=([|z|,\theta])^n$

$|z^n|=|z|^n$

$Arg(z^n)=n\,Arg(z)$

$(1+i)^9=a+ib$

$z=1+i \Rightarrow |z|=\sqrt{2}$

$z=\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)=\sqrt{2}\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}i\right)$

$$\sum_{i=1}^{10} 1 = \underbrace{1+1 \ldots +1}_{10 \text{ fois}}$$

$$\sum_{i=1}^{10} i = \frac{n(n+1)}{2} = \frac{10 \times 11}{2} = 55$$

$$\sum_{i=1}^{10} i^2 = \frac{n(n+1)(2n+6)}{6} = \frac{10 \times 11 \times 21}{6}$$

$= 385 \times 10 + 2 \times 55 \times 55 + 385 \times 10$

$= \boxed{13750}$

Ensa 2013

Reste $1234^{4321} + 4321^{1234}$ par 7

$4321 \,|\, \underline{7} \;\; 617 \quad R = 2$ $\qquad 1234 \,|\, \underline{7} \;\; 176 \quad R = 2$

$a = x7 + 2 \Rightarrow R = 2$

$b = y7 + 2$

M1 Le nombre $2^{100} - 1$ calculatrice

$2^{100} - 1 = \ldots\ldots$ | 3 et 31 oui

M2 : $2^1 - 1 = 1$ × $\quad 2^4 - 1 = 15$ ✓ ×

$2^2 - 1 = 3$ ✓ × $\quad 2^5 - 1 = 31$ × ✓

$2^3 - 1 = 7$ ✓ × $\ldots \quad 2^{10} - 1 = 1023$ ✓ ✓

$2^n - 1 / 3 \qquad n$ paire [illegible]

$2^n - 1 / 31 \qquad$ [illegible]

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$= [\sqrt{2}; \frac{\pi}{4}]$

$z^9 = [(\sqrt{2})^9; 9 \times \frac{\pi}{4}]$

$= [16\sqrt{2}; \frac{8\pi}{4} + \frac{\pi}{4}]$

$= [16\sqrt{2}; 2\pi + \frac{\pi}{4}] = [16\sqrt{2}; \frac{\pi}{4}]$

$z^9 = 16\sqrt{2}(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4})$

$= 16\sqrt{2}(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i) = 16 + 16i$

$\boxed{a = b = 16}$

$$(a+b)^n = \sum_{k=0}^{n} C_n^k a^k b^{n-k}$$

$$(a+b)^2 = \sum_{k=0}^{2} C_k^2 a^k b^{2-k}$$

$= C_0^2 a^0 b^2 + C_1^2 a^1 b^1 + C_2^2 a^2 b^0$

$= a^2 + 2ab + b^2$

$C_2^0 = \frac{A_0^2}{0!} = \frac{0!}{0!} = 1$

$C_2^1 = \frac{A_1^2}{1!} = \frac{2}{1} = 2$

$C_2^2 = \frac{A_2^2}{2!} = \frac{2 \times 1}{2 \times 1}$

$$\sum_{k=0}^{2016} (-1)^k \cdot C_{2016}^k \qquad \binom{n}{p} = C_p^n$$

$\left.\begin{array}{l} n = 2016 \\ b = -1 \\ a = 1 \end{array}\right\} (1-1)^{2016} = 0^{2016} = 0$

$$\sum_{j=1}^{1010} \sum_{i=1} (i+j)^2 = \sum\sum (i^2 + 2ij + j^2)$$

$= \sum\sum i^2 + \sum\sum 2ij + \sum\sum$ [illegible]

$= \sum i^2 \cdot \sum 1 + 2 \sum i \cdot \sum j + \sum j^2 \sum$ [illegible]

$\boxed{\text{Les astuces sur somme Arithmétique}}$

$$\sum_{i=1}^{10} = i=1 + i=2 \dots + i=10$$

$$= 1+1+1+\dots+1 = 10 \times 1 = 10$$

$$\sum_{i=1}^{10} k = k\sum_{i=1}^{10} 1 = k \times 10 = 10k$$

$$\sum_{i=2}^{5} = 5+5+5+5 = 20$$

$$\sum_{i=15}^{135} 1 = 1 \times (\text{عدد الحدود}) = 1 \times (135-15+1) = \boxed{121}$$

$$\boxed{\sum_{i=min}^{max} = max - min + 1}$$

$$\sum_{i=1}^{2} k\,i = k\sum_{i=1}^{2} i$$

$$\sum_{i=1}^{2} (i+j) = \sum i + \sum j$$

$$\sum i \cdot j = \sum i \cdot \sum j$$

$$\sum_{i=1}^{3} (2+15i) = \underbrace{(2+15)}_{i=1} + \underbrace{(2+15\times 2)}_{i=2} + \underbrace{(2+15\dots)}_{i=3}$$

$$= 17 + 32 + 47 \dots$$

$$\sum_{i=1}^{3} (2+15i) = \sum_{i=1}^{3} 2 + \sum_{i=1}^{3} 15i$$

$$= 2 \times (3-1+1) + 15\sum_{i=1}^{3} i$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$\boxed{\text{Somme des premiers entiers}}$

$$\sum_{i=1}^{\infty} = \lim_{n \to +\infty} \sum_{i=1}^{n}$$

$$1+2+3+\dots+5000 = ??$$

$$\sum_{i=1}^{5000} i =$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$S = 1+2+3\dots+n-1+n$$

$$S = n + n-1 + \dots + 2+1$$

$$2S = (1+n) + (\cancel{2}+n) + (1+n) + (n+1)$$

$$2S = (n+1)\overset{\text{عدد الحدود}}{(n-1+1)}$$

$$\boxed{S = \frac{n(n+1)}{2}}$$

Application:

$$1+2+\dots+5000 = \frac{5000 \times 5001}{2}$$

$$\sum_{i=1}^{10} i = \frac{10 \times 11}{2} = 55$$

Resumé:

$$\cancel{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

$$\sum_{i=1}^{n} k = k \times n$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\lim_{h \to 0} \frac{1}{h} \ln\left(1+\frac{h}{x}\right) = \frac{1}{h} \times \frac{h}{x} = \frac{1}{x}$$

13) $$\lim_{x \to 0} \frac{4^x - 2^x}{x} = \frac{e^{x\ln 4} - e^{x\ln 2}}{x}$$

$$= \frac{1 + x\ln 4 - 1 - x\ln 2}{x}$$

$$= \ln 4 - \ln 2$$

$$= \boxed{\ln 2}$$

14) $$\lim_{x \to +\infty} \frac{\sin(2x)}{3x} + \left(1+\frac{4}{x}\right)^x$$

$\lim_{x \to +\infty} \sin x$ ~~et~~ $\lim_{x \to +\infty} \cos x$

$$-1 \leqslant \sin 2x \leqslant 1$$

$$\frac{-1}{3x} \leqslant \frac{\sin 2x}{3x} \leqslant \frac{1}{3x}$$

$$\frac{\sin 2x}{3x} = 0$$

$$\lim \left(1+\frac{4}{x}\right)^x = e^{x\ln\left(1+\frac{4}{x}\right)}$$

$$= e^{x \times \frac{4}{x}} = \boxed{e^4}$$

$\lim \frac{3}{\ldots} + \frac{3}{\ldots} + \ldots + \frac{3}{n}$; $-3+4+\ldots+15=?$

① **Les astuces**

$$\lim_{x\to+\infty}\left(1+\frac{1}{\sqrt{x}}\right)^{-\sqrt{x}}$$

$$\lim_{x\to+\infty}\frac{1}{\left(1+\frac{1}{\sqrt{x}}\right)^{\sqrt{x}}}$$

$$\lim_{x\to+\infty}\frac{1}{\left(1+\frac{a}{x}\right)^{x}} = \frac{1}{e} = e^{-1}$$

$$\boxed{\lim_{x\to+\infty}\left(1+\frac{a}{x}\right)^{x} = e^{a}}$$ Astuce 1

قيمة y من خلال:

$\log_2(x) = 10$ و $\log_x(y) = 100$

Astuce 2: $\boxed{\log_a(x) = y \iff x = a^y}$

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

$$\log_x(y) = 100 \iff y = x^{100}$$

$$\log_2(x) = 10 \iff x = 2^{10}$$

$$\iff y = x^{100} \text{ و } x = 2^{10}$$

$$\iff y = (2^{10})^{100} = \boxed{2^{1000}}$$

$$S_n = \sum_{k=1}^{n} k C_n^k$$

قيمة المجموع:

$$S_n = \sum_{k=1}^{n} k . C_n^k$$

$$= 1C_n^1 + 2C_n^2 + 3C_n^3 \ldots nC_n^n$$

نعوض $n=3$:

$$S = 1C_3^1 + 2C_3^2 + 3C_3^3$$
$$= 12$$

نعوض n ب 3 في

$\boxed{A = n.2^{n-1}}$ $B = (n-1)2^n$; $C = n2^n$

$D = 2^n$ و $E = n.3^{n-1}$

$A = 3.2^{3-1} = 12$

$B = (3-1)2^3 = 16$

$C = 3.2^3 = 24$

$D = 2^3 = 8$

$E = 3.3^{3-1} = 27$

Astuce 3:
مجموع بدلالة n
نقوم بتعويض n
بأي قيمة

$$S_n = \sum_{k=1}^{n} \frac{1}{k(k+1)}$$

$\lim_{n\to+\infty} S_n$ نهاية S_n

$$\frac{1}{k} - \frac{1}{k+1} = \frac{1}{k(k+1)}$$

$$S_n = \sum_{k=1}^{n} \frac{1}{k(k+1)}$$

$$= \sum_{k=1}^{n} \frac{1}{k} - \frac{1}{k+1}$$

$$= 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} \ldots \frac{1}{n} - \frac{1}{n+1}$$

$$= 1 - \frac{1}{(n+1)}$$

$$\boxed{\lim_{n\to+\infty} S_n = 1}$$

Astuce 4: $\boxed{\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}}$

⑨ $\sum_{i=3}^{15} i = 3+4+\dots+15 = ?$

② $\ln\left(\left|\frac{x+1}{x-1}\right|\right)$ مجموعة تعريف

$* \ x \in D_f : \ \ln(x) \begin{cases} x \neq 0 \\ x \geq 0 \end{cases} \rightarrow x > 0$

$\frac{x+1}{x-1} > 0$ et $\frac{x+1}{x-1} \neq 0$

$\ln|U_n| \Leftrightarrow D_f$

$\sqrt{\ } > 0$ مع $U(x) \neq 0$ و $0 \neq$ مقام

$x \in D_f \Leftrightarrow \frac{x+1}{x-1} \neq 0$ و $x-1 \neq 0$

$\Leftrightarrow x+1 \neq 0$ و $x-1 \neq 0$

$\Leftrightarrow x \neq -1$ و $x \neq 1$

$D_f = \mathbb{R} - \{-1; 1\}$

مجموعة تعريف:

$f(x) = \ln\left(\left|1 - \frac{1}{\sqrt{x}}\right|\right)$

$x \in D_f \Leftrightarrow 1 - \frac{1}{\sqrt{x}} \neq 0$ و $x > 0$

$\Leftrightarrow 1 \neq \frac{1}{\sqrt{x}}$ و $x > 0$

$\Leftrightarrow x \neq 1$ و $x > 0$

$D_f =]0; 1[\cup]1; +\infty[$

نهاية $\lim_{n \to +\infty} \frac{\sin(4x)}{7x} + \left(1 + \frac{2}{n}\right)^n$

لا توجد $\left(\cancel{\frac{\sin +\infty}{\cos +\infty}}\right)$

$\lim_{n \to +\infty} \frac{\sin(4n)}{7n} + \lim \left(1 + \frac{2}{n}\right)^n = 0 + e^2$

$-1 \leq \sin(4n) \leq 1$

$\lim_{+\infty} = 0 \quad -\frac{1}{7n} \leq \frac{\sin(4n)}{7n} \leq \frac{1}{7n} \to 0$

Astuce: نهاية sin و cos لا توجد في +∞

الـ أطير

لتكن U_n متتالية حسابية بحيث

$U_4 = 0 \ ; \ U_6 = 1$

إذن قيمة الحد الأول U_1 هي:

$V_{n+1} - V_n = r$

$\forall n \in \mathbb{N} : V_n = V_p + (n-p)r$

$\frac{V_n - V_p}{n-p} = r \Leftrightarrow \frac{U_6 - U_4}{6-4} = -\frac{1}{2}$

$\boxed{r = -\frac{1}{2}}$

$U_1 = U_4 + (1-4) \times -\frac{1}{2} = \boxed{\frac{3}{2}}$

Astuce: متتالية حسابية

$\boxed{U_n = U_m + (n-m)r}$

U_n متتالية حسابية أساسها $r > 0$ و حدها الأول $V_0 = 1$ بحيث $10 = (V_4)^2 + (V_2)^2$

$\forall n \in \mathbb{N} ; \ V_n = V_p + (n-p)r$

$V_n = V_0 + nr$

$\boxed{V_n = 1 + nr}$

$(1 + 4r)^2 + (1 + 2r)^2 = 10$

$10 = 1 + 8r + 16r^2 + 1 + 4r + 4r^2$

$\Leftrightarrow 20r^2 + 12r - 8 = 0$

$\Leftrightarrow 5r^2 + 3r - 2 = 0$

نحدد Δ:

$r_1 = -1$ أو $r_2 = \frac{2}{5}$

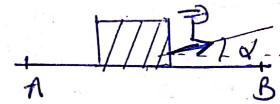
بما أن $r > 0$ فإن $r = 2/5$

② $\sum_{i=3}^{15} i = 3 + 4 \ldots + 15 = ?$

الميكانيك

① شغل قوة $\vec F$:

$W(\vec F) = \vec F \cdot \vec{AB} = F \cdot AB \cdot \cos(\vec F, \vec{AB})$ ①

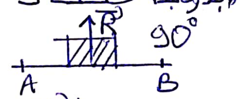


$W(\vec T) = \vec T \cdot \vec{AB}$

$= T \cdot AB \cdot \cos\alpha$

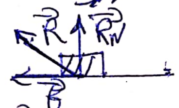
شغل قوة الاحتكاك

بدون احتكاك



$W(\vec R) = 0$

باحتكاك



$\vec R = \vec f + \vec R_N$

$W_{\vec R} = W_{\vec f} + W_{\vec R_N}$

$W_{\vec R} = W_{\vec f} = f \cdot AB \cdot \cos(\pi)$

$\boxed{W_{\vec R} = W_{\vec f} = -f \cdot AB}$

قوة الوزن $\vec P$

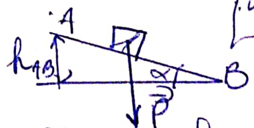
$W(\vec P) = mg\,h_{AB} = mg(z_f - z_i)$

أفقي

$h_{AB} = 0$

$W(\vec P) = mg\,h_{AB} = 0$

مائل بزاوية α



$W(\vec P) = \pm mg\,h_{AB}$

$\sin\alpha = \frac{h_{AB}}{AB}$

$h_{AB} = AB \cdot \sin\alpha$

$\boxed{W(\vec P) = mg\sin\alpha \cdot AB}$

② المسار دائري:

$\boxed{W(\vec P) = mgr}$

حالة ②

$\cos\theta = \frac{h}{r}$

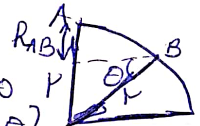
$h = \cos\theta \cdot r$

$h_{AB} = r - r\cos\theta$

$\boxed{h_{AB} = r(1-\cos\theta)}$

$\boxed{W(\vec P) = mgr(1-\cos\theta)}$

حالة ③:

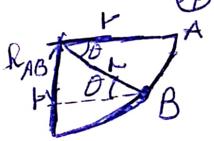


$h_{AB} = r - r\sin\theta$

$= r(1-\sin\theta)$

$\boxed{W(\vec P) = mgr(1-\sin\theta)}$

حالة ④:



$h_{AB} = \sin\theta \cdot r$

$\boxed{W(\vec P) = mg\sin\theta \cdot r}$

مبرهنة الطاقة الحركية:

$\Delta E_c = \sum W(\vec F)$ قوة ثابتة

$E_{c_f} - E_{c_i} = W(\vec P) + W(\vec R) + W(\vec T)$

$\frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = \pm mgh - f \cdot AB + T \cdot AB \cos(\vec T, \vec{AB})$

دوران

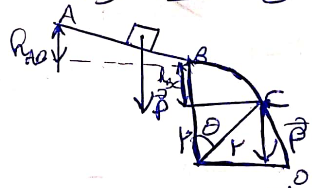
$W(\vec{F}) = M_{\vec{F}} . \Delta\theta$

$= \vec{F} . d . \Delta\theta$

$\Delta E_c = \sum W(\vec{F})$

$$\boxed{\frac{1}{2} J_\Delta . \theta_f^2 - \frac{1}{2} J_\Delta . \theta_i^2 = \sum M_{\vec{F}} \Delta\theta}$$

ينطلق جسم بدون سرعة بدائية على AB وبدون احتكاك



$$\frac{1}{2} m (V_B^2 - V_A^2) = W(\vec{P}) + W(\vec{R})$$

$$\frac{1}{2} m V_B^2 = P . AB . \sin\alpha = mg AB . \sin\alpha$$

$$\boxed{V_B = \sqrt{2g . AB . \sin\alpha}}$$

يتحرك على المسار (BCO) بدون احتكاك $W(\vec{R}) = 0$

تعبير V_C:

$$\frac{1}{2} m (V_C^2 - V_B^2) = W(\vec{P}) + W(\vec{R})$$

$W(\vec{P}) = mg . h_{BC}$

$= mg(r - r\cos\theta)$

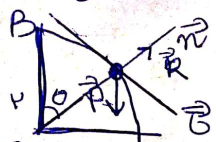
$= mgr(1 - \cos\theta)$

$$\frac{1}{2} \cancel{m} (V_C^2 - V_B^2) = \cancel{m} g r (1 - \cos\theta)$$

$$V_C^2 = V_B^2 + 2gr(1 - \cos\theta)$$

$$\boxed{V_C = \sqrt{V_B^2 + 2gr(1 - \cos\theta)}}$$

تعبير R بدلالة V_B و r و g:



P_j $\vec{t}$

$\vec{P} + \vec{R} = m\vec{a}_T$

$P_t + R_t = ma_t$

$mg . \sin\theta = m\frac{dV}{dt}$

اسقاط على $\vec{n}$

$P + R = m\vec{a}_n$

$P_n + R_N = m . a_N$

$mg\cos\theta - R = m\frac{V^2}{r}$

$P_n = mg\cos\theta$ $\quad a_N = \frac{V^2}{R}$

$\vec{R} - R$ $\quad a_T = \frac{dV}{dt}$

$$\boxed{R = mg\cos\theta - m\frac{V^2}{r}}$$

$$R_C = mg\cos\theta - m\frac{V_C^2}{r}$$

ونعلم أن $V_C = \sqrt{V_B^2 + 2gr(1-\cos\theta)}$

$$R_C = mg\cos\theta - m\frac{V_B^2 + 2mgr(1-\cos\theta)}{r}$$

$$R_C = m\left(g\cos\theta - 2g(1-\cos\theta) - \frac{V_B^2}{r}\right)$$

$$\boxed{R_C = m\left(3g\cos\theta - 2g - \frac{V_B^2}{r}\right)}$$